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ADAPTIVE CONTROL LAW DESIGN FOR MODEL UNCERTAINTY COMPENSATION

J. E. SORRELLS
DYNETICS, INC.
1000 EXPLORER BLVD.
HUNTSVILLE, AL 35806

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AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-6553

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This technical report has been reviewed and is approved for publication.

ANDREW G. SPARKS, 1st Lt, USAF
Project Engineer

DAVID K. BOWSER, Chief
Control Dynamics Branch
Flight Control Division

FOR THE COMMANDER

H. MAX DAVIS,
Assistant for Research and Technology
Flight Control Division
Flight Dynamics Laboratory

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19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Disturbance Accommodating Control principles were applied to the design of a baseline aerospace flight control system. The resulting DAC controller designs were evaluated in terms of their ability to maintain an ideal model response when the closed loop system was subjected to significant uncertainties. These uncertainties included parameter perturbations external disturbances, and unmodelled dynamics. The DAC designs were compared to other adaptive techniques, including Self-Tuning Regulator and Model Reference Adaptive Control concepts. These designs were also compared to a classical Proportional-Integral-Derivative controller. All of the adaptive controller designs were judged relative to each other and relative to the PID design, in terms of performance, design, complexity, and confidence of operation (reliability). In every case the controllers designed using Dynetics innovative approach were able to equal or surpass the STR and MRAC controllers in terms of performance robustness while preserving a linear time-invariant implementation structure.

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FOREWORD

This report summarizes work performed by Dynetics, Inc. under contract F33615-88-C-3609 for Wright Research and Development Center, Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio 45433-6553. This was a Phase I Small Business Innovative Research (SBIR) award. The work described was performed during the period August 1988 - January 1989.

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1.0 Introduction

Aerospace vehicles of the present and future are being designed to operate in extremely large flight envelopes in terms of the velocity, acceleration, altitude, and maneuverability that they are required to achieve. Not only are these vehicles required to operate in such large flight envelopes, but they must maintain specified performance margins while doing so. The development of such vehicles presents a great challenge to the flight control system designer. The lack of adequate analytical techniques and experimental data for systems expected to operate on the extremities of existing operational boundaries results in significant uncertainty concerning the dynamical description of the vehicle. Furthermore, aeroservoelastic effects arising from flexible vehicle characteristics will become more pronounced and unpredictable at higher velocities and accelerations. This means that the mathematical models currently used to describe vehicle motion and to design flight control systems may not be appropriate in terms of parametric and structural fidelity for many of the operating conditions of interest.

Adaptive feedback control has long been regarded as one of the leading choices for designing control laws for systems that are subject to the types of uncertainties described above. However, most of the existing adaptive control techniques have performance limitations and/or implementation drawbacks that forbid them from providing the operating robustness and/or operational confidence required of flight control systems for realistic vehicles. The primary goal of this Phase I effort was to develop a control system design methodology that would produce a control law capable of providing the necessary performance in the presence of uncertainties while avoiding the limitations and drawbacks of current adaptive control design techniques. Dynetics' innovative approach was to apply Disturbance Accommodating Control (DAC) theory to the flight control problem to create a novel alternative to traditional adaptive control system design.

Controllers that were designed using this innovative approach were compared to other controllers designed using various contemporary adaptive control theories. These designs were based upon Self-Tuning Regulator (STR) and Model Reference Adaptive Control (MRAC) concepts. All controller designs were evaluated in identical operating environments which included parameter variations, unmodeled dynamics, and external disturbances. The relative merits of each technique were compared in terms of performance robustness, design complexity, and operational confidence. In every case the controllers designed using Dynetics' innovative approach were able to equal or surpass the STR and MRAC controllers in terms of performance robustness while preserving a linear time-invariant implementation structure.

The development of a design methodology that will produce realizable control laws that can compensate for significant uncertainties is a contribution to the field of feedback control theory in general, and to the aerospace industry in particular. The feasibility of using DAC principles to develop such an adaptive control system design methodology was demonstrated with complete success in this Phase I effort. The applications of the resulting design methodology are as varied as the types of systems under development and consideration. All practical control systems are designed by using a mathematical model that has been simplified to a degree. Even in the most comprehensive cases there are invariably higher order effects that are deemed to be negligible. In this sense, all controller designs are subjected to varying degrees of uncertainty when implemented in realistic systems and therefore will benefit from the results of this study.

Evaluation of design complexity, confidence of operation and performance robustness verifies that the DAC based designs are superior to the other adaptive control techniques. The MRAC system exhibited very good performance characteristics for certain choices of tuning parameters, but the inherent nonlinearity of the resulting controller tends to limit the applicability of this method. The only tools available to analyze and measure the reliability and robustness of the closed loop system are repeated simulation and Liapanov stability analysis. Again, the STR approach resulted in a nonlinear closed loop system which was subject to the same analysis problems as the MRAC. Beyond the issue of nonlinearity is the fundamental limitation of the estimation of system parameters in the presence of external disturbances. The resulting DAC based controllers are linear time-invariant systems, which means that all of the standard linear analysis tools (Nyquist, Bode, root locus, etc.) are still applicable.

2.0 Objectives

The objectives of the Phase I study were as follows:

(1) Characterization of parameter and dynamical uncertainties

(2) Application of Disturbance Accommodating Control theory to development of control law design methodologies for systems subject to significant uncertainties

(3) Specification of candidate control algorithms

(4) Problem formulation and candidate solution development, and

(5) Performance demonstration.

The nature of the uncertainties that a control system will encounter can be characterized by considering three types of effects: parameter uncertainty, uncertain external disturbances, and unmodeled dynamics.

2.1 Characterization of Uncertainties

Parameter uncertainty can be mathematically described by consideration of the general state-space system equation shown in Figure 2.1. Parameter uncertainty is manifested by perturbations away from some expected (nominal) values of the elements of the A, B, and C system matrices. These perturbations ΔA, ΔB, and ΔC also correspond to the uncertainty associated with the coefficients of the differential equations which govern the behavior of the system when expressed in a transfer function form as shown in Figure 2.2.

It is interesting to note the correspondence between the state-space system representation and the transfer function representation in terms of the numerator and denominator coefficients and their respective locations in the A,B,C system matrices. The transfer function of Figure 2.2 can be written in several different canonical state-space forms, each of which has a distinct configuration. In most cases the denominator coefficients will appear as elements of the A matrix (either in a single row, a single column, or some combination of each). The numerator coefficients, however, can appear as entries in the C matrix alone, in the B matrix alone, or as some combination of the two.

On the surface it appears that the choice of a canonical form is completely arbitrary and therefore should not affect the controller design process. This is in fact the case when all of the parameters of the system are certain or known. However, if the parameters are subject to perturbations or

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \\ \dot{x}3 \\ \cdot \\ \cdot \\ \dot{x}n \end{bmatrix} = \begin{bmatrix} a11 & a12 & a13 & \ldots & a1n \\ a21 & a22 & a23 & \ldots & a2n \\ a31 & a32 & a33 & \ldots & a3n \\ \cdot & \cdot & \cdot & & \\ & & & & \\ an1 & an2 & an3 & \ldots & ann \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ \\ \\ xn \end{bmatrix} + \begin{bmatrix} b1 \\ b2 \\ b3 \\ \\ \\ bn \end{bmatrix} u$$

ΔA ΔB

$$y = \begin{bmatrix} c1 & c2 & c3 & \ldots & cn \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ \cdot \\ \cdot \\ xn \end{bmatrix}$$

ΔC

Figure 2.1. **Parameter Uncertainty, State-Space Representation**

Numerator Uncertainty

$$\frac{Y(s)}{U(s)} = \frac{s^m + a_{m-1}s^{m-1} + \ldots + a_1 s + a_0}{s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}$$

Denominator Uncertainty

Figure 2.2. **Parameter Uncertainty, Transfer Function Representation**

uncertainties, it is very important to understand how the uncertainty will affect the system response. This in turn dictates that the actual state-space description of the system should have some relative significance and therefore should not be chosen arbitrarily. More specifically, the B-matrix describes a mapping of the system input into the particular state-space described by the A-matrix, in a geometrical sense. And conversely, the C-matrix describes the mapping of the entire state vector into some observed sub-space. Or stated more heuristically, the B-matrix describes how the input gets distributed into the system, and the C-matrix describes how the system motion gets described in the form of observed outputs.

Another type of uncertainty is external disturbances. External disturbances are defined in the context of this study as those inputs which are both directly unmeasurable and uncontrollable by the control system designer. External disturbances are mathematically described by the addition of forcing terms in the system equations as shown in Figure 2.3. These disturbances can also be described in transfer function or block diagram form by the configuration shown in Figure 2.4.

The third type of uncertainty considered in this study was unmodeled dynamics. Unmodeled dynamics can be described mathematically by the system equations shown in Figure 2.5. The resulting effect of the unmodeled dynamics is that the dimension of the system model used to design the control system is less than the dimension of the actual system to be controlled. Unmodeled dynamics can be further divided into two categories: additive and multiplicative, as shown in Figure 2.6. The effect in both cases is similar in that each type increases the dimension of the actual system, but their treatment in the design of a control system is very different.

2.2 General Problem Statement

One of the original motivations for this study was to develop a control law design methodology for hypervelocity vehicles, therefore the class of problems chosen for consideration was taken to be representative of these types of aerospace vehicles. A benchmark design problem was formulated to capture the important features of designing a realistic flight control system for such a vehicle.

The particular problem formulated was to design a controller for the longitudinal motion of an aerospace vehicle. This problem was used as the baseline test problem throughout this Phase I study. A linearized dynamical model describing the vehicle's motion in terms of angular rate about some center of gravity reference was developed. The transfer function which characterizes this dynamical model is shown in Figure 2.7. This dynamical model is very common and is found to reoccur frequently in a multitude of aerospace control problems. This particular model describes the pitch or yaw

$$\dot{x} = Ax + Bu + Fw + \ldots$$

$$y = Cx + Hw + \ldots$$

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \\ \dot{x}3 \\ \cdot \\ \cdot \\ \dot{x}n \end{bmatrix} = \begin{bmatrix} a11 & a12 & a13 & \ldots & a1n \\ a21 & a22 & a23 & \ldots & a2n \\ a31 & a32 & a33 & \ldots & a3n \\ \cdot & \cdot & \cdot & & \\ \cdot & & & & \\ an1 & an2 & an3 & \ldots & ann \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ \\ \\ xn \end{bmatrix} + \begin{bmatrix} b1 \\ b2 \\ b3 \\ \\ \\ bn \end{bmatrix} u + \begin{bmatrix} f1 \\ f2 \\ f3 \\ \cdot \\ \cdot \\ fn \end{bmatrix} w$$

$$y = \begin{bmatrix} c1 & c2 & c3 & \ldots & cn \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ \cdot \\ \cdot \\ xn \end{bmatrix} + \begin{bmatrix} h1 \\ h2 \\ \cdot \\ \cdot \\ \cdot \\ hn \end{bmatrix} w$$

Figure 2.3. External Disturbances, State-Space Format

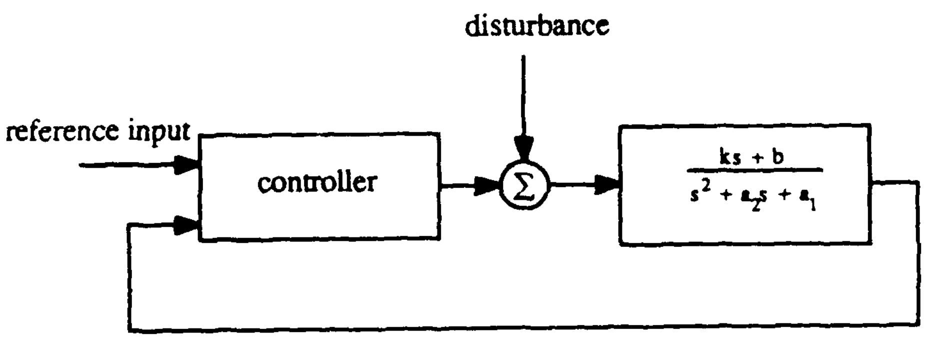


Figure 2.4. External Disturbances, Transfer Function Format

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

- Controller Designed Based upon Second Order Dynamics Assumption

- Higher Order Terms Contribute Significantly to System Response

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \\ \dot{x}3 \\ \cdot \\ \cdot \\ \dot{x}n \end{bmatrix} = \begin{bmatrix} a11 & a12 & a13 & \ldots & a1n \\ a21 & a22 & a23 & \ldots & a2n \\ a31 & a32 & a33 & \ldots & a3n \\ \cdot & \cdot & \cdot & & \\ \cdot & & & & \\ an1 & an2 & an3 & \ldots & ann \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ \\ \\ xn \end{bmatrix} + \begin{bmatrix} b1 \\ b2 \\ b3 \\ \\ \\ bn \end{bmatrix} u$$

$$y = \begin{bmatrix} c1 & c2 & c3 & \ldots & cn \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ \cdot \\ \cdot \\ xn \end{bmatrix}$$

Figure 2.5. Unmodeled Dynamics

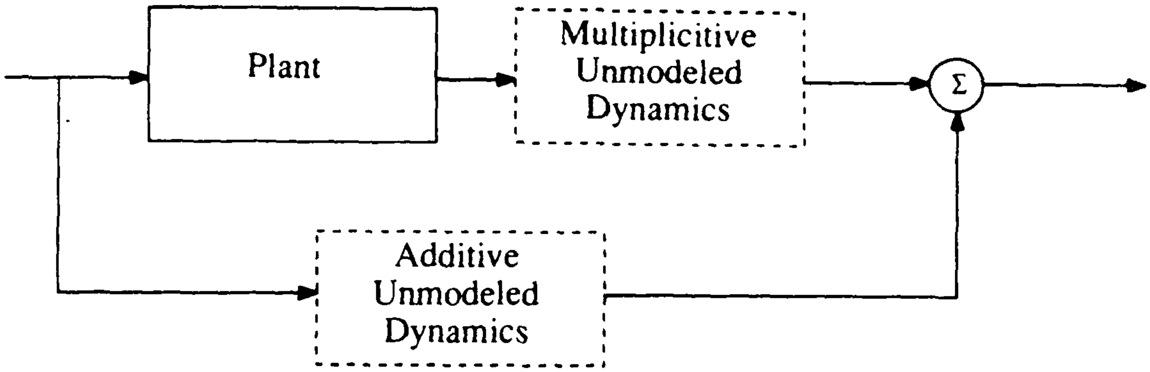


Figure 2.6. Two Catagories of Unmodeled Dynamics

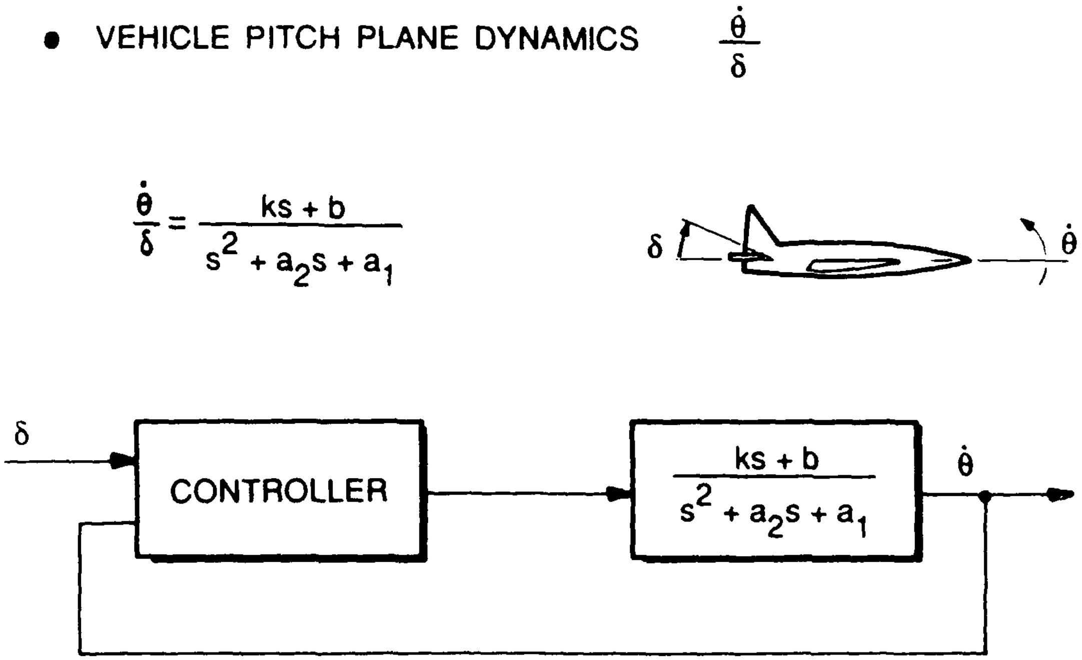


Figure 2.7. Dynamical Description of Baseline Problem

rate for aircraft, missile angular turning rate (either tail or canard controlled), and space launch vehicle angular turning rate (atmospheric portion of flight). The use of this particular dynamical problem makes the results of this Phase I study very general and applicable to a variety of systems.

The benchmark problem was to design a suitable controller which could maintain a set of specified performance requirements in the presence of the uncertainties described in section 2.1.

3.0 Performance versus Stability

A succinct distinction is made between performance and stability for the controller designs considered in this study. For our purposes stability is defined in the traditional manner by describing the controllers ability to drive the homogeneous error system to zero asymptotically. Performance is defined in terms of the controllers ability to force the system output to reproduce a desired transient shape and to maintain specified input command tracking margins.

This distinction between stability and performance is necessary due to the fact that almost all of the research in adaptive control for the last three decades has been preoccupied with proving global stability for certain classes of plant uncertainty. Engineers concerned with the application of adaptive control techniques are much more concerned with the controllers ability to maintain performance specifications in the presence of uncertainties which may be unknown exactly but which are invariably bounded in realistic problems. For this reason, the benefits and limitations of various adaptive control designs may not be at all apparent to the flight control system designer.

In the area of flight control, performance is typically the ultimate justification for using any type of active feedback control. The airframes encountered are almost invariably statically stable, and only in very recent history have there been any actual systems built which rely on active stability augmentation for operation. Active control is employed on virtually all vehicles as a means of enhancing the performance of the closed loop system. In the case of piloted aircraft, performance translates into the specific handling qualities of the vehicle which are required by the pilot. For a missile, the flight control system performance can be directly measured in terms of the control system contribution to overall miss distance. For a space launch vehicle, performance is specified in terms of the control systems ability to maintain robustness margins due to the fundamental importance of achieving mission success. In each of these realistic cases, it is obvious that stability is assumed and that performance is really the primary concern.

An example of a specific performance template which might describe the required closed-loop pitch-rate response for a piloted aircraft is shown in Figure 3.1. Both the transient and tracking requirements are apparent from the specified regions of acceptable and unacceptable performance. This specification is typical of the type of handling quality requirement available in several Military Standard documents for piloted aircraft.

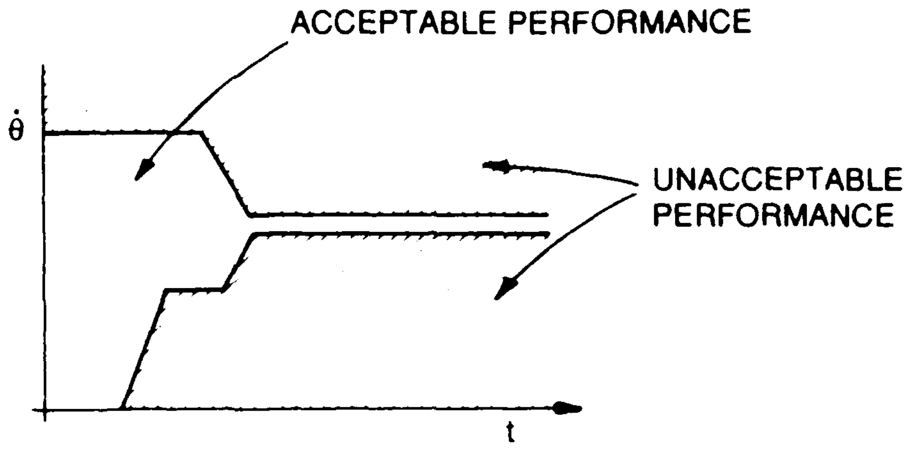


Figure 3.1. Typical Performance Boundaries

4.0 Disturbance Accommodating Control

Disturbance Accommodating Control (DAC) theory has existed as a design discipline since the late 1960's [1]. Concepts and techniques for the characterization and design of disturbance accommodating controllers have been refined and extended into a fairly complete technology [2], [3], [4].

The original motivation for the development of DAC was to generalize the disturbance compensation techniques of classical control, such as integral feedback, feedforward compensation, and notch filtering, into the framework of multivariable systems. Figure 4.1 illustrates a typical closed loop system consisting of a controller whose purpose is to force the plant to follow a servo-command in the presence of plant uncertainty, external disturbances, and corrupt measurements. Disturbances are defined as all of those plant and controller external inputs or internal actions that are totally unexpected and uncontrollable from the designers point of view.

Disturbances can be generally classified into two broad categories: erratic (noise-like) disturbances and waveform-like disturbances (See Figure 4.2.) Erratic noise signals are best described using statistical properties such as mean and variance. Waveform type disturbances are typically non-ergodic in nature and therefore their statistical properties are not useful in terms of designing a compensating control strategy. A specific example of this is the case of a constant random disturbance. If an ensemble average is constructed, the mean value could well be zero, but if a temporal average is calculated the mean value is simply the value of the disturbance itself. By taking a temporal average the mean value with respect to time is calculated, which in the case of a random constant is simply the value of the constant. An ensemble average is calculated by taking a cross-section of the entire possible set of random constants at a particular instance in time, which gives the mean value of the sample set as opposed to the mean value of a particular sample. In this case knowledge of the statistical properties of the disturbance are not useful for designing a control strategy to offset the effects of the constant disturbance. In fact it is well known that integral feedback is the proper compensation scheme for unknown constant random disturbances.

The central theme of DAC theory is in modeling disturbance processes as time signals which have particular "waveform" characteristics (See Figure 4.3.) These types of signals can always be mathematically described by the following type of equation

$$\omega(t) = c_1 f_1(t) + c_2 f_2(t) + \ldots\ldots + c_m f_m(t)$$

This is a function-space representation of

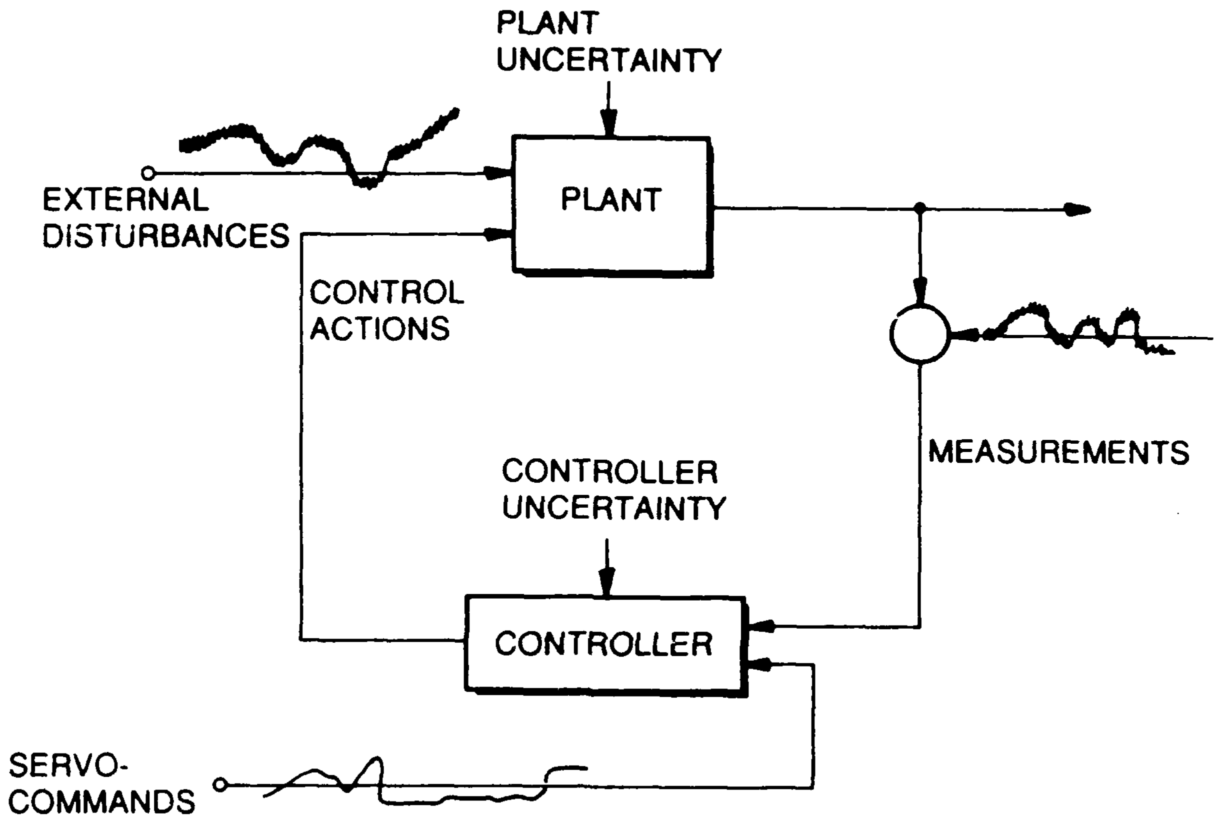
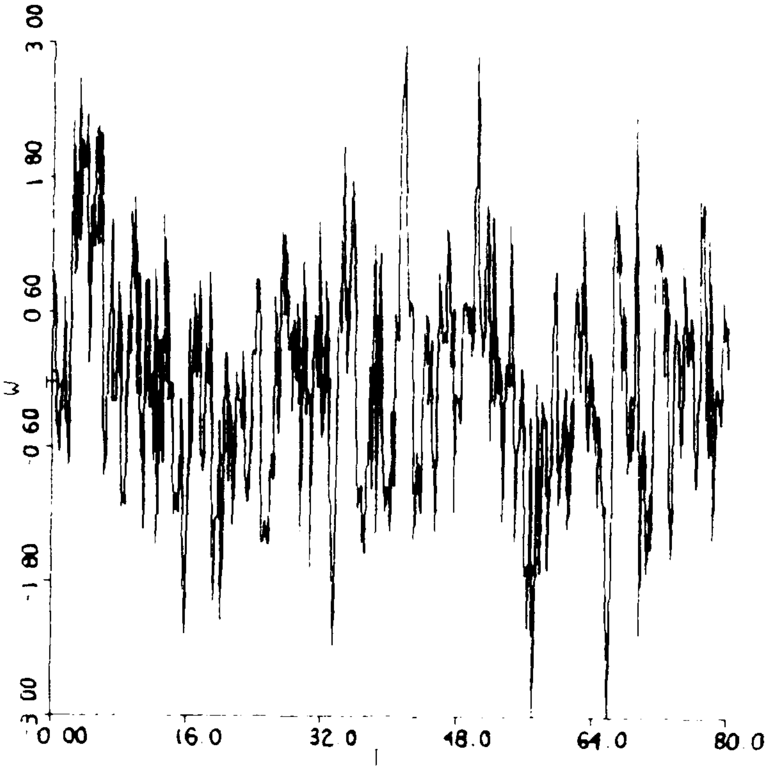
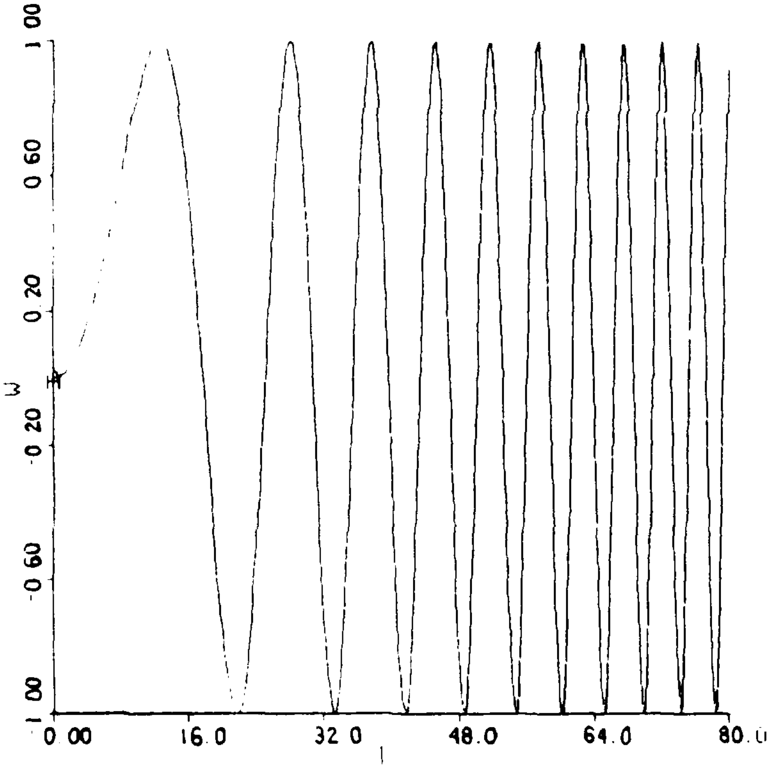


Figure 4.1. Block Diagram of Closed Loop System



• Wide band white noise input



• Single sine wave input, frequency swept from 0 - 8 rad/sec

Figure 4.2. Classification of Disturbance Types

$\omega(t)$ where the basis set $\{f_1, f_2, \ldots, f_m\}$ is chosen to reflect the natural waveform patterns inherent in the disturbance behavior. These functions f_i embody the character of the disturbance signal. The choice of the particular types of functions to be used as the basis set is strictly a design alternative. These functions could be constants, sinusoids, exponentials, polynomials, etc. The c_i's are constants which are chosen once the basis set is defined. The particular values used for these constants specifies the exact time characteristic of the disturbance model. This is, in effect, a generalized spline-function model of the disturbance.

Although DAC was initially developed as a means of compensating for external disturbances, it has also been suggested as a means of compensating for other system uncertainties such as parameter perturbations. Consider the following dynamical system

$$x = Ax + Bu + Fw$$

$$y = Cx$$

Parameter uncertainty is represented by perturbations in the A, B, and C system matrices, away from their nominal or expected values. If the nominal values are written as A_n, B_n, and C_n and the perturbations are δA, δB, and δC, then the system equations can be rewritten as

$$\dot{x} = (A_n + \delta A)x + (B_n + \delta B)u + Fw$$

$$\dot{y} = (C_n + \delta C)x$$

and the uncertainties can be grouped as

$$x = A_n x + B_n u + (\delta A x) + (\delta B u) + Fw$$

$$y = C_n x + (\delta C x)$$

The basic philosophy of the DAC compensation technique is to treat all of the uncertainties shown in the system equations above as some type of undesirable disturbance (either internal or external). Following standard DAC design procedures, these uncertainties are treated as the output of a dynamical process. The next step in the design procedure is to construct a state-space model of the disturbance process.

Let

$$\dot{w} = c_1 f_1 + c_2 f_2 + \dots + c_m f_m$$

In standard DAC practice the disturbance model is constructed by first choosing a set of basis functions for the spline model which characterize w. In most cases the designer will have little if any knowledge concerning the waveform characteristics of the process w, and therefore would have no rational way to assign a set of effective basis functions to the spline model. In this case, practice has shown it to be very effective to use polynomials (linear, quadratic, cubic, etc.) for the basis functions. This follows from similar practices in the area of approximation theory. Quadratic and cubic splines can be fit to waveform types of signals to high degrees of accuracy over specified intervals of interest.

The state-space model for w is constructed by solving the inverse problem from differential equation theory, i.e. find the differential equation to which w is a solution. Once the differential equation in w is formulated, then the state-space model of this differential equation represents the state model of the disturbance process. As an example consider a case when very little is known about the actual uncertainty and the designer chooses to employ a quadratic spline to characterize w. Then

$$w = c_1 + c_2 t + c_3 t^2$$

which is the general solution to the following differential equation

$$\dddot{w} = 0$$

which can be represented by the following state-space model

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$w = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

Or, rewritten in general matrix format

$$z = Dz$$

$$w = Hz$$

In the case that nothing at all is known about the uncertainty, the polynomial spline approach is always a valid design solution. But for the case of δA perturbations in particular, we see that the natural choice for the basis functions of the spline are simply the eigenfunctions of the nominal system [4]. This choice of basis functions has been called the Adaptive DAC (ADAC) approach. The distinction between choices of basis functions and the design alternatives available are shown graphically in Figure 4.4.

In either case the design proceeds in similar fashion once the basis functions have been chosen. The basic idea in compensating for the uncertainty characterized by z is to identify it's effect and to cancel it (there are other possible strategies such as minimization of uncertainty or possible utilization of uncertainty [2], but only cancellation will be considered here). To cancel the effects of z it must first be identified in some manner. In DAC this identification is achieved by means of a composite state observer constructed by augmenting the original system with the dynamical model of z. Specifically, recall

$$x = A_n x + Bu + Fw$$

$$y = Cx$$

and

$$z = Dz$$

$$w = Hz$$

then

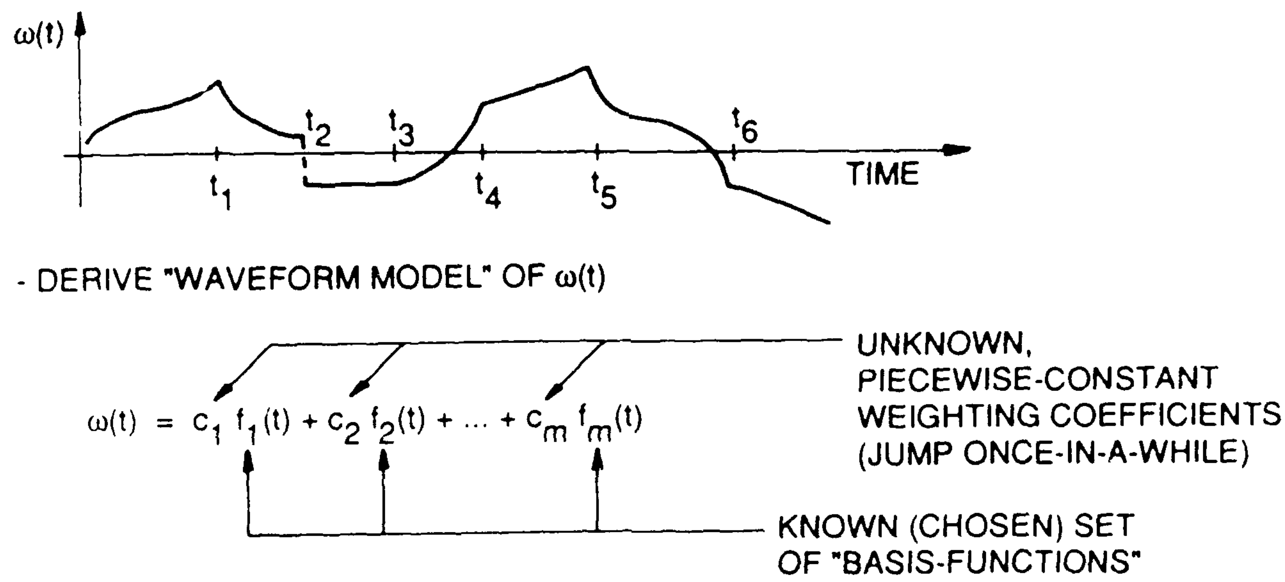


Figure 4.3. Disturbance With Waveform Characteristics

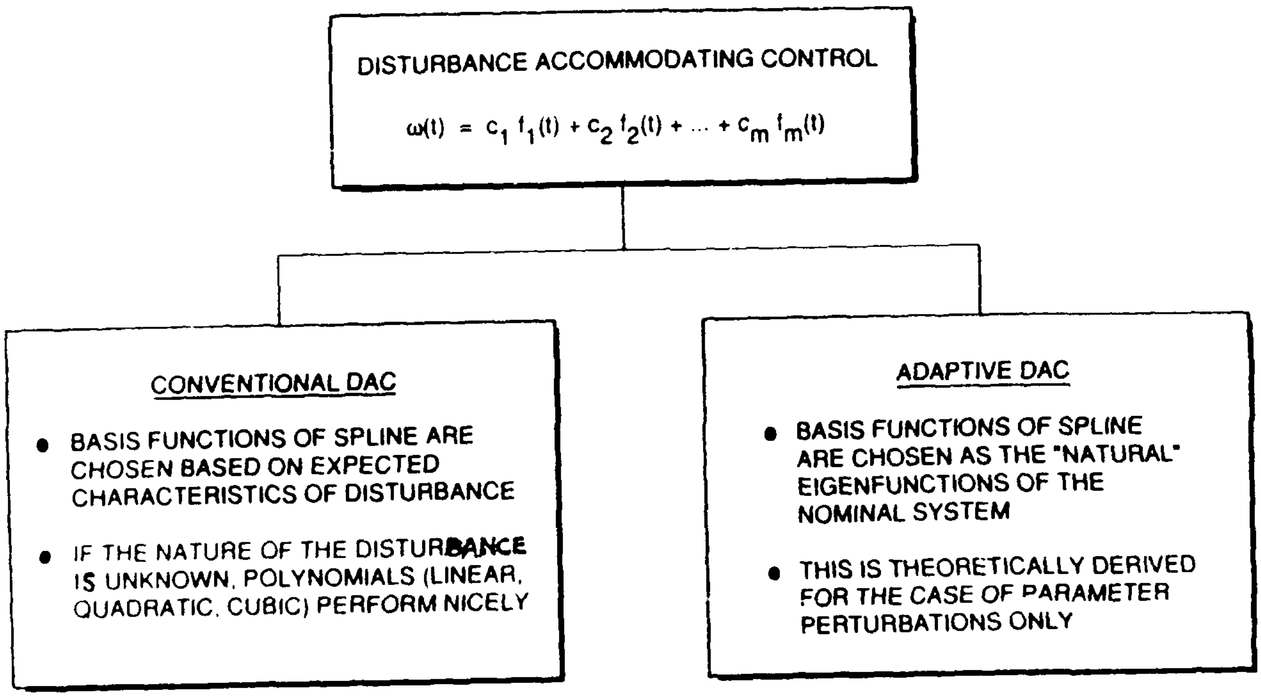


Figure 4.4. Disturbance Accommodating Control Design Alternatives

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_n & FH \\ 0 & D \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

$$y = [C \quad 0] \begin{bmatrix} x \\ z \end{bmatrix}$$

This state-space system is simply another linear system for which a composite observer can be generated. The composite observer can be written as

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{z}} \end{bmatrix} = \begin{bmatrix} A_n & FH \\ 0 & D \end{bmatrix} \begin{bmatrix} \hat{x} \\ \hat{z} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} k_{ox} \\ k_{oz} \end{bmatrix} \left[y - [C \quad 0] \begin{bmatrix} \hat{x} \\ \hat{z} \end{bmatrix} \right]$$

A block diagram of the composite observer is shown in Figure 4.5.

4.1 Adaptive DAC Design for Baseline Problem

For specific examples of the design procedure, again consider the benchmark problem outlined in section 2.2. The plant is given by

$$\frac{y}{u} = \frac{ks + b}{s^2 + a_2 s + a_1}$$

which corresponds to the following differential equation

$$\dot{y} + a_2 \dot{y} + a_1 y = ku + bu$$

This system can be written in the following state-space format

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} k \\ a_2 k - b \end{bmatrix} u$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Now assume that the objective is to track some constant input y_{sp}. The

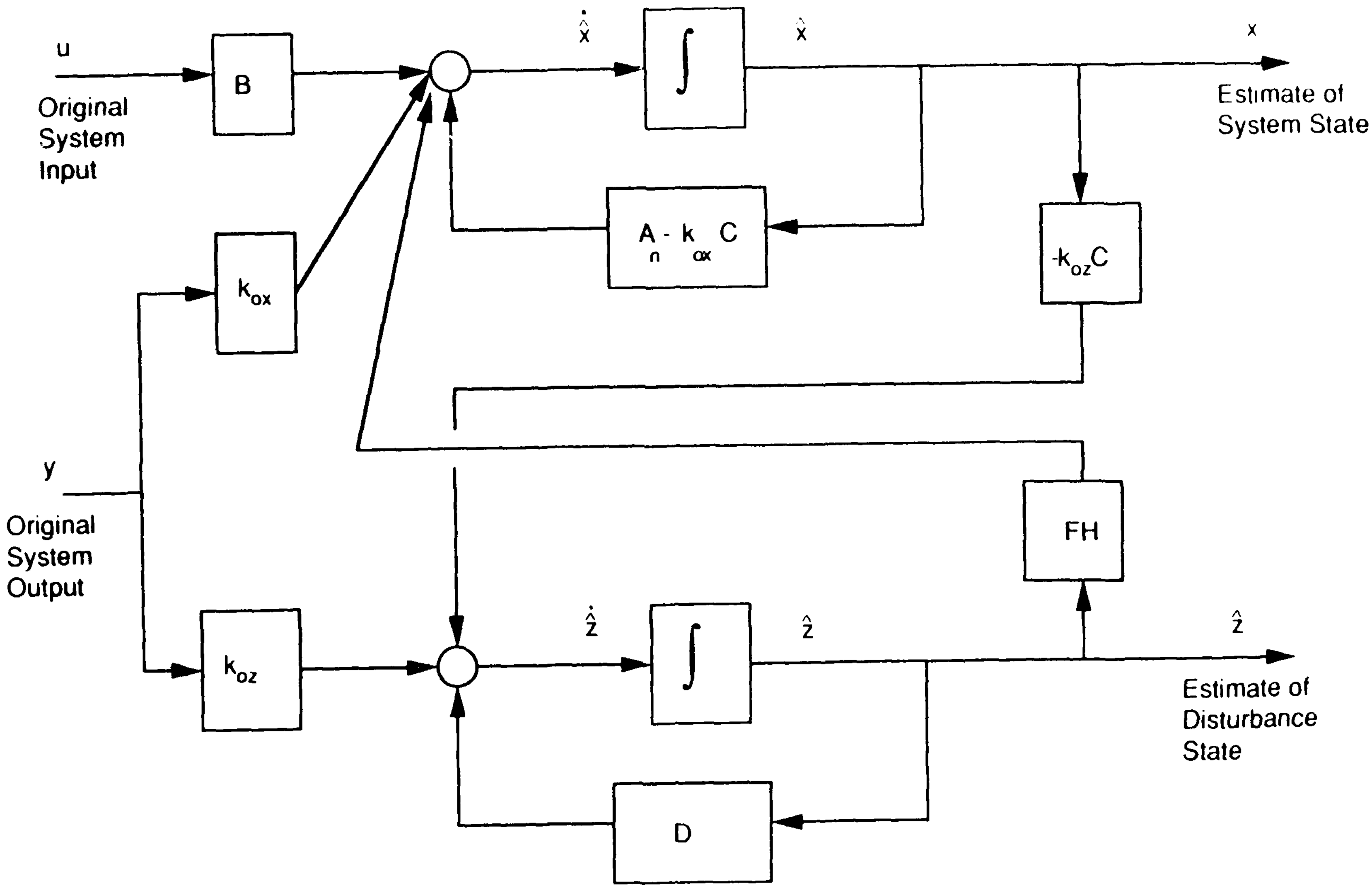


Figure 4.5. Block Diagram of Composite State Observer

following error can then be defined

$$\varepsilon = y_{sp} - y$$

$$y = y_{sp} - \varepsilon$$

and the following error system is written

$$-\ddot{\varepsilon} - a_2\dot{\varepsilon} - a_1\varepsilon + a_1 y_{sp} = k\dot{u} + bu$$

$$\ddot{\varepsilon} + a_2\varepsilon + a_1\varepsilon = a_1 y_{sp} - k\dot{u} - bu$$

If the plant parameters are perturbed from their nominal values

$$a_1 = (a_{1n} + \delta a_1)$$

$$a_2 = (a_{2n} + \delta a_2)$$

then the error system becomes

$$\ddot{\varepsilon} + a_2\dot{\varepsilon} + a_1\varepsilon = a_1 y_{sp} - \delta a_2\dot{\varepsilon} - \delta a_1\varepsilon - k\dot{u} - bu$$

Now let

$$z_a = -(\delta a_2\dot{\varepsilon} + \delta a_1\varepsilon)$$

$$z_d = k\dot{u} + (\text{any other disturbance terms})$$

where z_a represents an effective "disturbance" due only to the effects of parameter variations, and z_d represents a disturbance which contains all other external and internal uncertainty effects.
then

$$\ddot{\varepsilon} + a_2\dot{\varepsilon} + a_1\varepsilon = a_1 y_{sp} - bu + z_a - z_d$$

Recall that the objective is to force y to follow a constant input, or to force ε to zero in a prescribed manner. Examination of the error system equation as written above indicates exactly the control law necessary to force ε to zero, namely

$$u = u_p + u_a + u_d$$

where

$$u_p = \frac{a_{1n}}{b_n} y_{sp}$$

$$u_a = \frac{1}{b_n} z_a$$

$$u_d = -\left(\frac{1}{b_n}\right) z_d$$

The control law is separated into three parts u_p, u_a, and u_d. The primary control signal u_p is designed to achieve the fundamental objective of tracking the set-point command. This part of the control signal is designed assuming that the plant is free of disturbances, and is chosen exactly the same as in other state-space based design approaches such as pole placement. In this particular problem no pole placement was necessary due to the fact that the nominal system characteristic equation is exactly the same as the desired closed-loop characteristic equation. The signal u_a is chosen to cancel the effects of parameter variations in the closed-loop system. The signal u_d is designed to cancel the effects of all other internal and external uncertainties. Due to the fact that the composite observer and therefore the resulting compensator are linear systems, the property of superposition is valid which allows the design of the control law to be separated into three parts which can be performed independently. One of the fundamental attributes of DAC design is the explicit identification of quantities such as z_d and z_a which facilitates the type of control law design shown above.

Now all that is needed to implement this control law are the estimates of z_a and z_d. For this the composite state observer is necessary. First the state models of the uncertainties must be constructed. Since the term z_d represents all of the uncertainty effects other than parameter variations, the specific character of this signal is unknown. Therefore, we will follow conventional DAC techniques discussed earlier and will choose a linear polynomial spline model for z_d. This choice of basis functions is a very good general model for completely unknown types of signals. The corresponding state model is

$$z_d = 0$$

$$z_d = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z_{d1} \\ z_{d2} \end{bmatrix}$$

$$\begin{bmatrix} \dot{z}_{d1} \\ \dot{z}_{d2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z_{d1} \\ z_{d2} \end{bmatrix}$$

Now following the ADAC procedure, assume that the perturbation effects are characterized by

$$\ddot{z}_a + a_{n2} z_a + a_{n1} z_a = 0$$

$$z_a = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z_{a1} \\ z_{a2} \end{bmatrix}$$

$$\begin{bmatrix} \dot{z}_{a1} \\ \dot{z}_{a2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_{n1} & -a_{n2} \end{bmatrix} \begin{bmatrix} z_{a1} \\ z_{a2} \end{bmatrix}$$

The choice of the nominal system parameters as the appropriate estimation matrix in the equations above follows from the development in [4], where it was suggested that the evolution of parameter variation effects on the nominal system could best be characterized by the eigenfunctions of the original nominal system. Now the complete composite system is formed

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dot{z}_{a1} \\ \dot{z}_{a2} \\ \dot{z}_{d1} \\ z_{d2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -a_{n1} & -a_{n2} & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -a_{n1} & -a_{n2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ z_{a1} \\ z_{a2} \\ z_{d1} \\ z_{d2} \end{bmatrix} +$$

$$\begin{bmatrix} -k_n \\ a_{n2}k_n - b_n \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

An observer is constructed for this composite system which is given again by

$$\dot{\hat{x}} = \mathcal{A}\hat{x} + \mathcal{B}u + k_o[y - \mathcal{C}\hat{x}]$$

where

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -a_{n1} & -a_{n2} & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -a_{n1} & -a_{n2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathcal{B} = \begin{bmatrix} -k_n \\ a_{n2}k_n - b_n \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathcal{C} = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

This observer will produce the estimates of z_a and z_d necessary to implement the control law, which in its final form is

$$u = u_p + u_a + u_d = \frac{a_{n1}}{b_n} y_{sp} + \left(\frac{1}{b_n}\right)\hat{z}_{a1} - \left(\frac{1}{b_n}\right)\hat{z}_{d1}$$

4.2 Conventional DAC Design

A conventional DAC controller design can be constructed using a similar procedure to that outlined above. As discussed earlier, the distinction between the conventional DAC and Adaptive DAC designs lies with the choice of basis functions used for the spline model of the uncertainty. In order to better understand the complete capabilities of the ADAC approach, a comparison with conventional DAC design methods was deemed to be useful.

Again consider the plant described in the baseline problem

$$\frac{y}{u} = \frac{ks + b}{s^2 + a_2 s + a_1}$$

which can be represented by the following state-space system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} b & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Now assume that all of the uncertainties can be lumped into one term z, which can be characterized by the following cubic polynomial

$$z = c_1 + c_2 t + c_3 t^2 + c_4 t^3$$

In this case we have assumed that <u>all</u> uncertainty effects, <u>including those due to parameter variations</u>, can be represented by the term z. Since this term includes effects from several different internal and external sources, it is assumed that little is known about the specific character of this signal. Therefore, as discussed earlier, a good choice for the basis functions of this disturbance model is a polynomial. This polynomial satisfies the following general differential equation

$$\ddddot{z} = 0$$

which has the state-space model

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

$$z = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

Recall the composite observer for the system is

$$\dot{\hat{x}} = \mathcal{A}\hat{x} + \mathcal{B}u + k_o[y - \mathcal{C}\hat{x}]$$

where

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -a_{n1} & -a_{n2} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathcal{B} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathcal{C} = \begin{bmatrix} b_n & k_n & 0 & 0 & 0 & 0 \end{bmatrix}$$

The control law for this case is simply

$$u = \left(\frac{1}{b_n}\right) y_{sp} - \hat{z}_1$$

5.0 Self-Tuning Regulators

The concept of self-tuning regulator adaptive controller design is much as the title would indicate. First an appropriate dynamical model for the actual system is developed, secondly some control law design algorithm such as pole placement or linear-quadratic-regulation (LQR) is chosen, and lastly the parameters of the dynamical model are estimated and used to solve for the required closed loop control law.

5.1 Regulator Design

There are several alternatives available for designing the regulator portion of the STR controller. One particular technique which is very straightforward is the polynomial approach to pole placement. Given the plant

$$A(s)y = B(s)u$$

where A(s), B(s) are polynomials in s, y is the plant output, and u is the plant input. The general control equation can be written as

$$L(s)u = -P(s)y + H(s)y_{ref}\,.$$

Substitution of the first equation into the second equation above results in

$$[A(s)L(s) + B(s)P(s)]y = B(s)H(s)y_{ref}$$

which can be written in transfer function form as

$$\frac{y}{y_{ref}} = \frac{B(s)H(s)}{A(s)L(s) + B(s)P(s)}$$

For the benchmark problem we have

$$A(s) = s^2 + a_2 s + a_1$$

$$B(s) = ks + b$$

and for the regulator design we choose

$$L(s) = s + k_1$$

$$P(s) = k_2 s + k_3$$

Then

$$[A(s)L(s) + B(s)P(s)] = (s^2 + a_2 s + a_1)(s + k_1) + (ks + b)(k_2 s + k_3)$$
$$= s^3 + (a_2 + k_1 + k_2 k)s^2 + (a_1 + k_1 a_2 + k_3 k + k_2 b)s + (k_1 a_1 + k_3)$$

The next step in the design process is to specify a characteristic polynomial which exhibits the transient behavior desired from the closed loop system. This characteristic polynomial has the following form

$$s^3 + a_{3d}s^2 + a_{2d}s + a_{1d}$$

Now we equate the coefficients of the last two equations above and solve for the controller gains. The result is written as

$$k_1 = [b^2(a_{3d} - a_2) - kb(a_{2d} - a_1) + a_{1d}k^2]/\Delta$$
$$k_2 = [(a_2 - a_{3d})(ba_2 - ka_1) + b(a_{2d} - a_1) - a_{1d}k]/\Delta$$
$$k_3 = [a_1 b(a_2 - a_{3d}) + a_1 k(a_{2d} - a_1) + a_{1d}(b - ka_2)]/\Delta$$
$$\Delta = b^2 - k(a_2 b - ka_1)$$

The final step is to estimate the plant parameters a_1, a_2, k, and b, choose the values a_{3d}, a_{2d}, a_{1d}, solve for k_1, k_2, k_3 and the problem is solved.

This specific procedure outlines the polynomial approach to pole placement, but this is by no means the only option available to the designer. Other popular design algorithms such as LQR could just as easily be used in this kind of problem.

5.2 Estimation

To implement the controller of section 5.1, we must obtain the estimated values of the plant. Most of the current literature on self-tuning regulator design encourages the separation and optimization of the regulator and estimator functions individually. Several alternative algorithms are available for estimation which are found in references such as [5] and [6].

Most estimation algorithms are based upon minimizing some function which measures the suitability of the estimate based on observed data. The two common measures of suitability are the square of the error between the estimate and the actual value which results in the class of least squares (LS) estimators, and the maximum likelihood (ML) class of estimators generated by the minimization of a probabilistic likelihood function. For all practical considerations the difference between the resulting algorithms for both types of measure functions is negligible.

Estimation can be performed in two basic modes, batch and recursive. In the batch mode a certain amount of input and output data is collected and processed in an "off-line" manner to generate the required estimates. In this mode it is possible to perform the estimation in either the time domain or in the frequency domain. The second mode of estimation is the recursive mode. When data is processed recursively a new estimate is produced each time a successive sample of input and output data is introduced into the estimator. The recursive mode can be thought of as an "on-line" mode of operation. For the type of systems considered in this study it is necessary that the estimates be available in an "on-line" manner since the nature of flight control problems is that the system under control is constantly changing. Therefore it is computationaly impossible to perform any kind of batch estimation using typical avionics hardware systems. For this reason only recursive estimation algorithms were considered in this study.

Several commonly available Recursive Least Squares (RLS) estimation algorithms were evaluated [7], [8], [9]. The nature of these algorithms makes them particularly amenable to implementation as discrete-time difference equations. This necessitates the use of a discrete-time model for the plant as described in the baseline problem.

Given the continuous time plant

$$\frac{y}{y_{ref}} = \frac{ks + b}{s^2 + a_2 s + a_1}$$

a corresponding discrete-time model is given by

$$\frac{y}{y_{ref}} = \frac{\bar{k}z^{-1} + \bar{b}z^{-2}}{1 + \bar{a}_2 z^{-1} + \bar{a}_1 z^{-2}}$$

The system can then be written in a recursive fashion as a difference equation

$$y(n) + \bar{a}_2 y(n-1) + \bar{a}_1 y(n-2) = \bar{k} y_{ref}(n-1) + \bar{b} y_{ref}(n-2)$$

$$y(n) = -\bar{a}_2 y(n-1) - \bar{a}_1 y(n-2) + \bar{k} y_{ref}(n-1) + \bar{b} y_{ref}(n-2)$$

$$y(n) = \phi^T \theta$$

where

$$\phi^T = y(n-1), y(n-2), y_{ref}(n-1), y_{ref}(n-2)$$

$$\theta^T = \left[-\bar{a}_2, -\bar{a}_1, \bar{k}, \bar{b}\right]$$

The vector ϕ is a regression vector of the inputs and outputs of the system. The vector θ is a parameter vector which contains the system parameters (coefficients of the open-loop system transfer function). We can now generate a new system defined by

$$\theta(n+1) = \theta(n)$$

$$y(n) = \phi^T(n)\,\theta(n)$$

where θ, ϕ are exactly as before.

The appropriate RLS estimator can simply be derived as a Kalman filter for the system above. The corresponding Kalman filter equations are

$$\hat{\theta}(n) = \hat{\theta}(n-1) + K_e(n)[y(n) - \phi^T(n)\hat{\theta}(n-1)]$$

$$K_e(n) = P(n)\phi(n) = P(n-1)\phi(n)[\lambda I + \phi^T(n)P(n-1)\phi(n)]^{-1}$$

$$P(n) = [I - K_e(n)\phi^T(n)]P(n-1)/\lambda$$

again, where ϕ is a vector of inputs and outputs and θ is the vector of parameters which are to be estimated. The second and third equations above give the optimal gain K_e of this estimator in terms of the covariance matrix P, both of which are calculated accordingly. The value λ is defined as the exponential "forgetting factor." This forgetting factor is used to discount older data which becomes invalid if the parameters vary as a function of time. For $\lambda = 1$ no data is discarded, as λ approaches 0 older data is discounted more quickly.

The use of a discrete-time plant model in the estimation algorithm brings along with it a subtle but very important point, and that is the actual values being estimated are the coefficients of the discrete-time transfer function and not the continuous time transfer function. The effect of transforming a continuous time system into a discrete time model is the mathematical equivalent of mapping the entire complex plane into the unit circle, as is readily recalled from elementary digital control theory (see Figure 5.1). The net result of this operation appears in the accuracy of the estimates achievable by any of the estimator algorithms. Since such a very large area is compressed into the unit circle, it is immediately obvious that estimating the parameters to within, say 10 percent, of their actual values in the discrete domain certainly does not mean that their values are known to within 10 percent in the continuous domain.

As a further example of this problem consider the general continuous time characteristic equation

$$(s + p_1)(s + p_2)..........(s + p_n)$$

which has a corresponding discrete-time equivalent (using the direct pole-mapping procedure outlined in [10])

$$(z - e^{-p_1 T})(z - e^{-p_2 T}) ..... (z - e^{-p_n T})$$

where T is the sampling period. Notice that as T approaches 0 (or the faster the system is sampled), that e^{-pT} approaches 1, and the discrete-time characteristic equation approaches

$$(1 - z^{-1})^n$$

which results in all of the roots of the characteristic equation becoming concentrated about the point (-1,0) in the discrete domain. This means that as the system is sampled faster that the discrete-time characteristic equation will tend towards

$$(1 - z^{-1})^n$$

regardless of the actual underlying continuous time characteristic equation which the discrete time model represents.

The practical implication of this result is that if very fast sample rates are to be used, then the number of significant digits in the estimates required will be quite large. As an example consider the second order continuous time characteristic equation

$$s^2 + 2s + 1$$

which has the corresponding discrete-time model

$$1 - (e^{-T} + e^{-2T})z^{-1} + (e^{-3T})z^{-2}$$

For the following sample periods T, this equation becomes

$$1 - 0.5z^{-1} + 0.05z^{-2}, \quad T = 1.0$$

$$1 - 1.72z^{-1} + 0.74z^{-2}, \quad T = 0.1$$

$$1 - 1.98z^{-1} + 0.98z^{-2}, \quad T = 0.01$$

The net result is that as the system is sampled faster, the discrete-time coefficients become less distinguishable. This is a totally counterintuitive result and is one that is not at all apparent to designers who are not intimately familiar with discrete recursive parameter estimation or STR adaptive control design.

Fortunately there is an alternative to performing estimation on discrete-time models of continuous systems, and that is to estimate the continuous time parameters directly. The delta operator method of [9] is exactly suited for this application and can be implemented using the same estimator routines discussed earlier. Using the delta operator approach it is true that the faster the system is sampled the more accurate the estimates become, which is the desired effect of fast sampling. In fact, it is not unusual to be able to estimate the continuous time parameters to within 1 percent of their nominal values under suitable conditions.

In view of all of the preceding discussion, the primary limitations of STR adaptive control can be summarized by considering two very important cases. The first limitation is the need for persistency of excitation in order to perform any kind of parameter estimation. Consider again the system equation

$$y(n) = -a_m y(n-1) \ldots - a_1 y(n-m) + b_m y_{ref}(n-1) \ldots + b_1 y_{ref}(n-m)$$

If the input $y_{ref} = 0$, then it is obvious that no information about $b_1, b_2, \ldots, b_m$ will be available. Also, if $y_{ref} = 0$ and the system is at rest ($y = 0$), then no information concerning $a_1, a_2, \ldots, a_m$ will be available either. This results in the requirement imposed throughout STR design literature that the input must be "sufficiently rich" in terms of excitation before STR adaptive control can be successfully employed.

A second and more debilitating limitation of STR adaptive control is the inability of any STR algorithm to cope with unmeasurable external disturbances which are similar in frequency content to the reference input. This can be easily demonstrated by considering the first-order example shown in Figure 5.2. The reference input is u, w is an unmeasurable external disturbance, and a is the plant parameter which has a nominal value a = 1 Take for example the case when u = 1 and the output y is observed to be the value y = 2. There is no way to discern from this limited set of input and output data whether the variation in y from it's nominal value of 1 to the actual value of 2, is due to a perturbation in a (i.e. the actual value of a might be 0.5 as opposed to the nominal expected value of 1), or whether in fact an external disturbance of w = 1 exists. The fundamental problem is that there is one equation which has two unknowns. One could arbitrarily postulate that certain observed output effects are caused by either the external disturbance or the parameter perturbation, but there is no way to uniquely determine a cause and effect relationship. Most of the current STR design algorithms attempt to circumvent this problem by isolating the input and output signals from the external disturbances in frequency by band-pass filtering these signals appropriately, but there is no cure for the case when the external disturbance has a similar frequency content to the input signal.

As an example consider the following system

$$\frac{y}{y_{ref}} = \frac{ks + b}{s^2 + a_2 s + a_1}.$$

where the nominal values of a_1 and a_2 are 1.0 and 2.0 respectively. Figure 5.3 shows the estimates of the parameters when no disturbance is present (w = 0), and Figure 5.4 shows the estimates produced by the exact same algorithm when a constant external disturbance is present (w = 1). The estimator used in this case was a RLS with exponential forgetting type. It is immediately obvious that the presence of an external disturbance severely limits the performance of the parameter estimator.

In view of the limitations and design considerations associated with STR adaptive control as generally outlined in this report, it is a logical conclusion that this particular method of adaptive control is not at all suitable for the type of realistic problems embodied by the benchmark problem considered herein. This in no way suggests that STR adaptive control does not have merit as a viable design technique, but it does strongly indicate that such an approach is inappropriate for the type of aerospace problems of interest in this study.

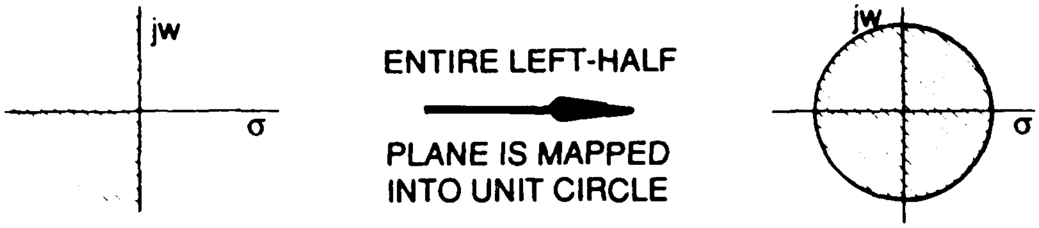


Figure 5.1. Mapping of Continuous Domain into Discrete Domain

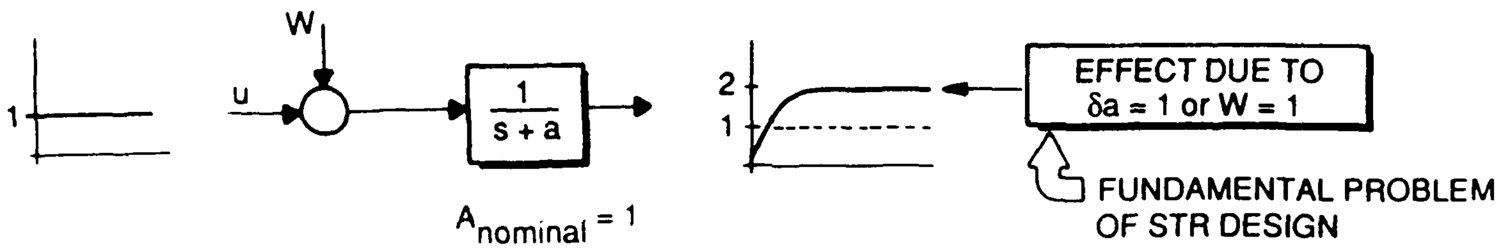
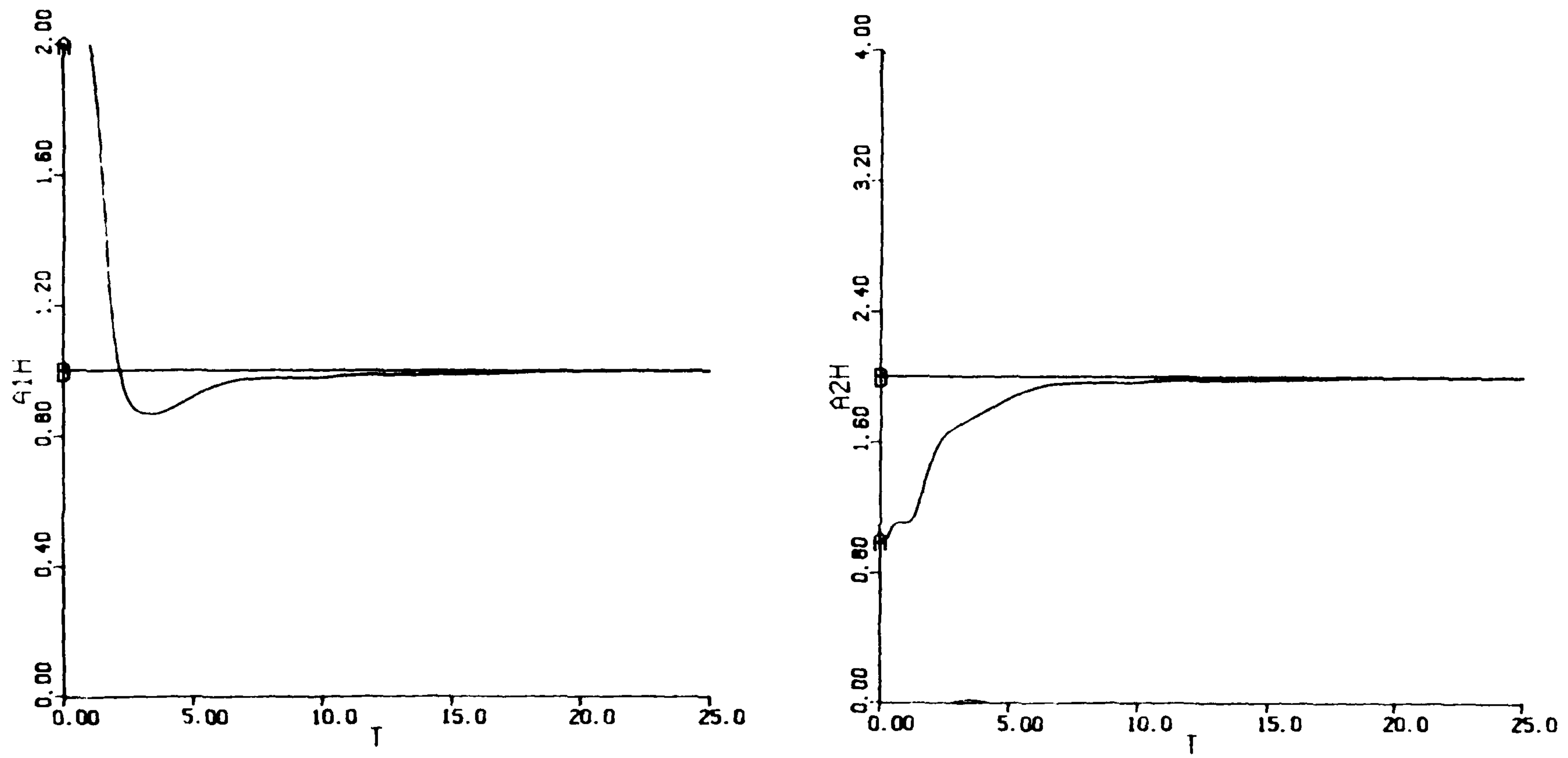
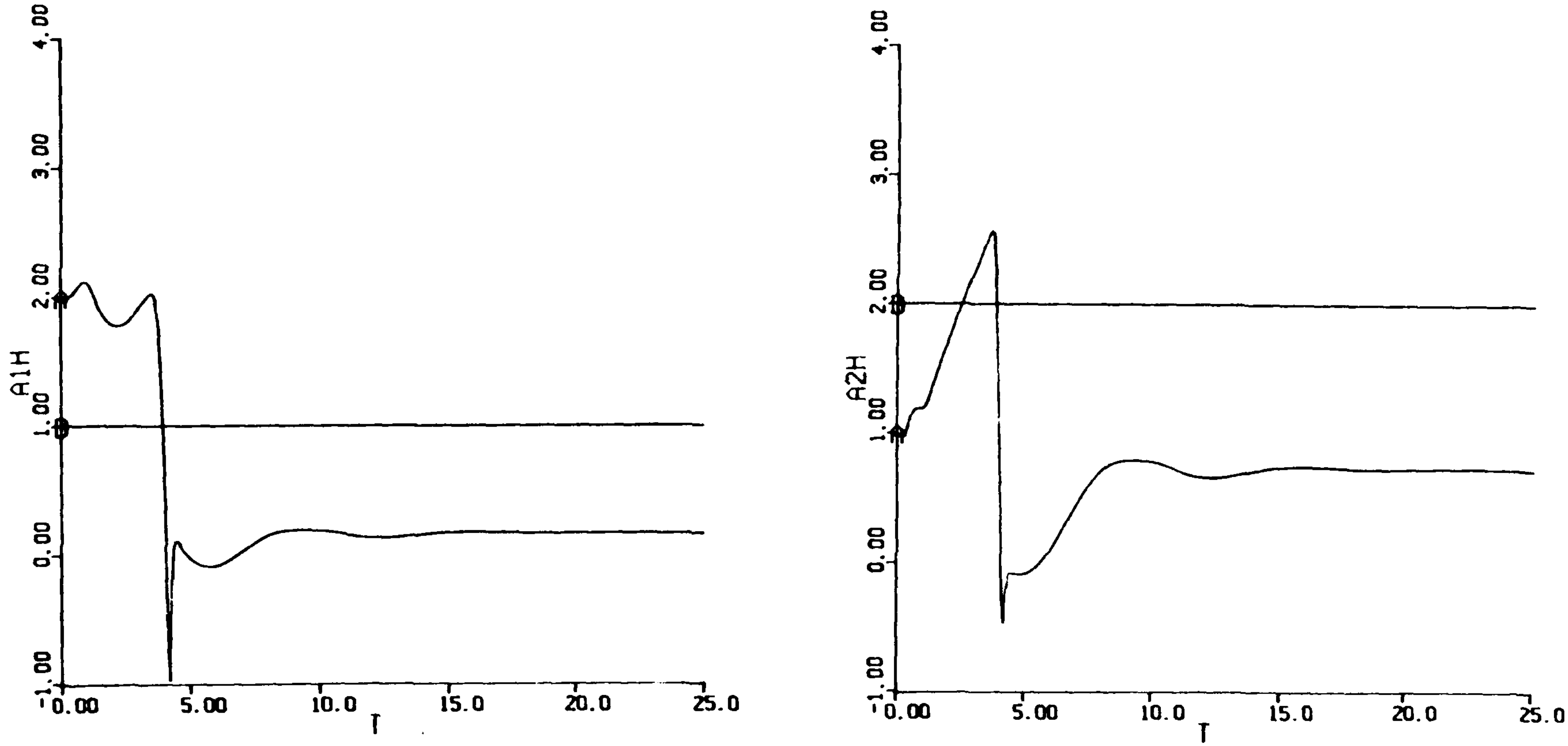


Figure 5.2. Effects of External Disturbances on Simple Estimation Problems



- Denominator coefficients, a1 & a2
- No external disturbance
- Pulse wave input, period = 20 sec, duration = 10 sec

Figure 5.3. Continuous Time Parameter Estimation, No External Disturbances



- Denominator coefficients, a1 & a2
- Constant external disturbance, magnitude = 1.0
- Pulse wave input, period = 20 sec, duration = 10 sec

Figure 5.4. Continuous Time Parameter Estimation, With External Disturbances

6.0 Model Reference Adaptive Control

The second major area of adaptive control theory is known as Model Reference Adaptive Control (MRAC). Implementation is achieved by constructing an ideal model which exhibits the desired response to some command reference, then comparing the actual plant output to the ideal model output to obtain an error signal. The error signal is used to modulate the feedback gains of the controller so as to eventually drive the difference between the ideal response and the actual response to zero (see Figure 6.1).

The origins of this type of control strategy can be traced back to the "MIT rule" developed at the Massachusetts Institute of Technology in the late 1950's. The MIT rule was a straightforward approach that suggested that the controller gains should be changed in proportion to the gradient of the error signal. Very simply put, the larger the error the larger the gain required, and the smaller the error the smaller the gain required. There have been many improvements and iterations made upon the basic concept over the last three decades, but most of the current MRAC techniques still contain these same basic concepts.

A detailed discussion of the specific MRAC design algorithm used in this study can be found in [11]. Sample applications are discussed in [12] and [13]. An outline of the controller design development is summarized in the following discussion.

Given a reference model system

$$\dot{x}_m = A_m x_m + B_m u_m$$

$$y_m = C_m x_m$$

and the plant equations

$$\dot{x}_p = A_p x_p + B_p u_p$$

$$y_p = C_p x_p$$

Define the error as

$$e = y_m - y_p$$

A reference vector is constructed

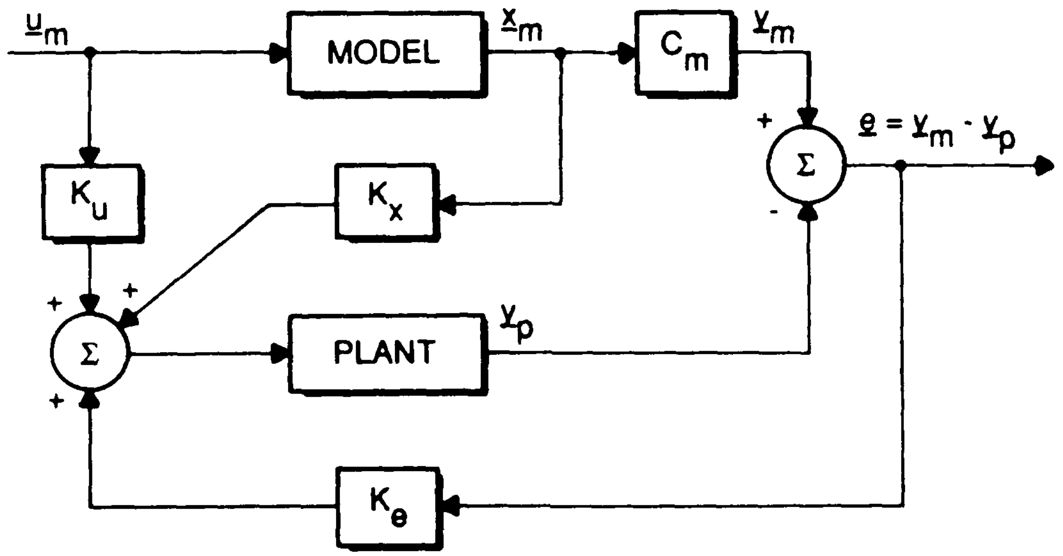


Figure 6.1. Block Diagram of MRAC System

$$r = \begin{bmatrix} e \\ x_m \\ u_m \end{bmatrix}$$

where e is the error as defined above, x_m is the ideal model state vector, and u_m is the reference input (possibly a vector). The adaptive control law is given by

$$u = k_a r$$

where

$$k_a = k_p + k_I$$

and

$$k_p = er^T T = e[e, x_m, u_m]T$$

$$k_I = er^T \bar{T} = e[e, x_m, u_m]\bar{T}$$

and the gain k_a can be rewritten as

$$k_a = [k_e, k_x, k_u] = k_p + \int k_I$$

where

$$k_e = e^2 T + \int e^2 \bar{T}\, dt$$

$$k_x = ex_m T + \int ex_m \bar{T}\, dt$$

$$k_u = eu_m T + \int eu_m \bar{T}\, dt$$

The adaptive control law can then be written as

$$u = ek_e + k_x x_m + k_u u_m$$

As a specific example consider the baseline problem which gives the following plant model

$$\frac{y}{y_{ref}} = \frac{ks + b}{s^2 + a_2 s + a_1}$$

This plant can be written in the following state-space canonical form

$$\dot{x}_1 = -a_2 x_1 + x_2 + k u_p$$

$$\dot{x}_2 = -a_1 x_1 + b u_p$$

$$y = x_1$$

The reference model for this system represents the desired closed loop response in transfer function form, and is given by

$$\frac{y}{u_m} = \frac{0.5s + 1}{s^2 + 2s + 1}$$

which can be written in state-space format as

$$\dot{x}_{m1} = -2x_{m1} + x_{m2} + 0.5u_m$$

$$\dot{x}_{m2} = -x_{m1} + u_m$$

$$y_m = x_{m1}$$

Now define

$$e = y_m - y$$

and the reference vector

$$r = \begin{bmatrix} e \\ x_{m1} \\ x_{m2} \\ u_m \end{bmatrix}$$

and

$$u = k_a r$$

where

$$k_a = k_p + k_I$$

where

$$k_p = er^T T = e[e, x_{m1}, x_{m2}, u_m]T$$
$$k_I = er^T \bar{T} = e[e, x_{m1}, x_{m2}, u_m]\bar{T}$$

then

$$k_a = [k_e, k_{x_1}, k_{x_2}, k_u]$$

$$k_e = e^2 T + \int e^2 \bar{T}\, dt$$
$$k_{x_1} = e x_{m1} T + \int e x_{m1} \bar{T}\, dt$$
$$k_{x_2} = e x_{m2} T + \int e x_{m2} \bar{T}\, dt$$
$$k_u = e u_m T + \int e u_m \bar{T}\, dt$$

Finally, the complete adaptive control law is

$$u = e k_e + k_{x_1} x_{m1} + k_{x_2} x_{m2} + k_u u_m$$

A block diagram of the complete closed loop system is shown in Figure 6.2.

T and $\bar{T}$ are weighting matrices to be chosen by the designer. The particular choice of these parameters will determine the adaptation rate and the domain of adaptability achievable by the controller. There are no analytical procedures available to aid in the selection of these weighting matrices, even though they are critical in determining the operation of the closed loop controller. The most straightforward method of choosing these weighting matrices is through an empirical study of observed ideal model following performance for specific choices of T and $\bar{T}$.

This particular adaptive control scheme exhibits very good performance characteristics in terms of maintaining the ideal response when the actual plant is subject to significant uncertainties. The plots shown in Figures 6.3, 6.4, 6.5, and 6.6 illustrate the performance capabilities of the MRAC controller as a function of the choice of T and $\bar{T}$. In each case the reference model is described by the reference plant above, with values k = 0.5, b = 1.0, $a_1 = 1.0$, $a_2 = 2.0$. The actual plant has parameters k = 0.5, b = 1.0, $a_1 = 1.0$, and $a_2 = 0.0$. Notice that the expected (or nominal) plant parameters are the values given by the ideal model description. The actual value of a_2 differs

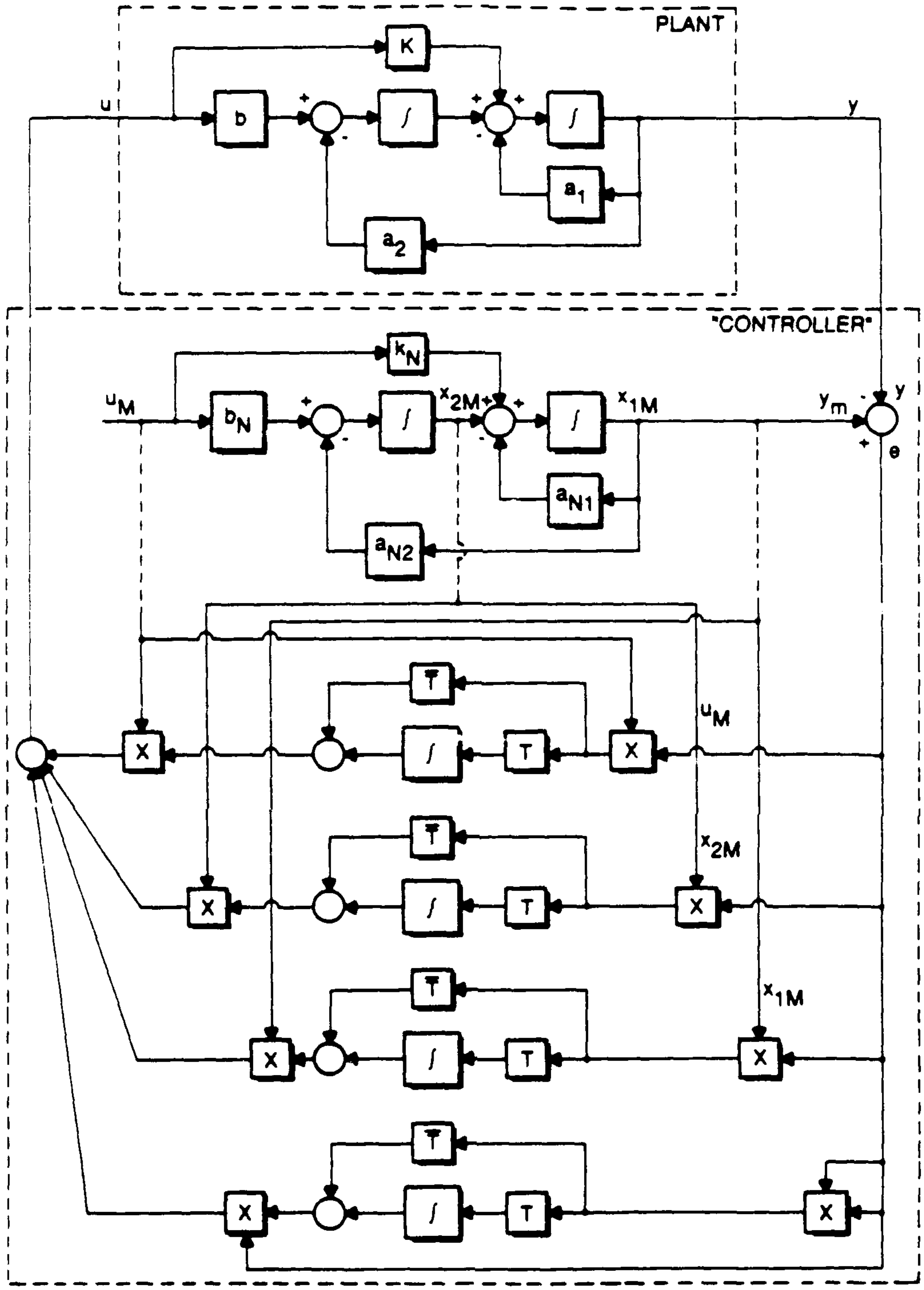


Figure 6.2. MRAC Controller Structure

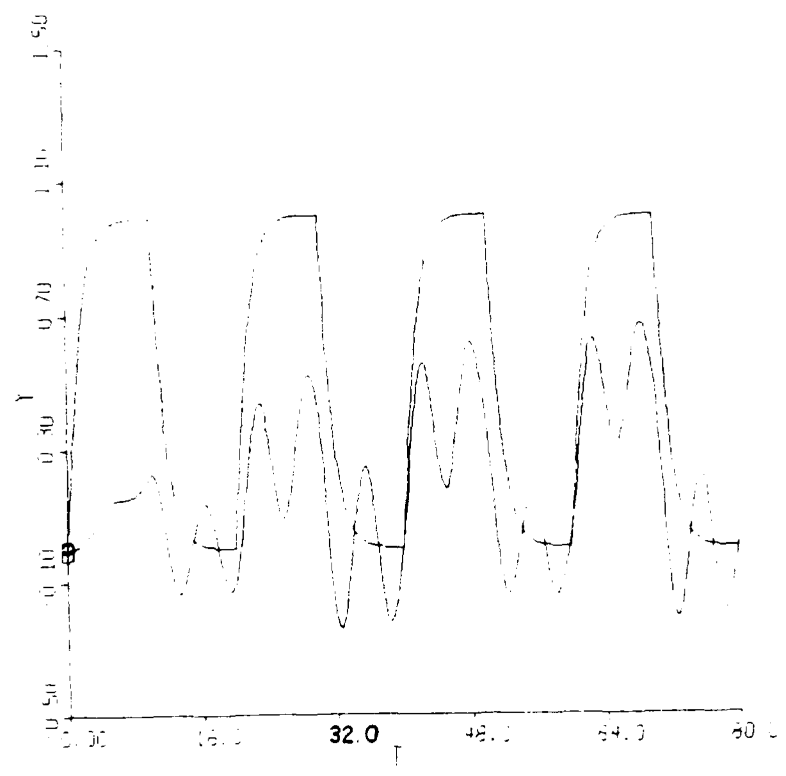


Figure 6.3. MRAC Controller Performance, T,T̄ = C.01

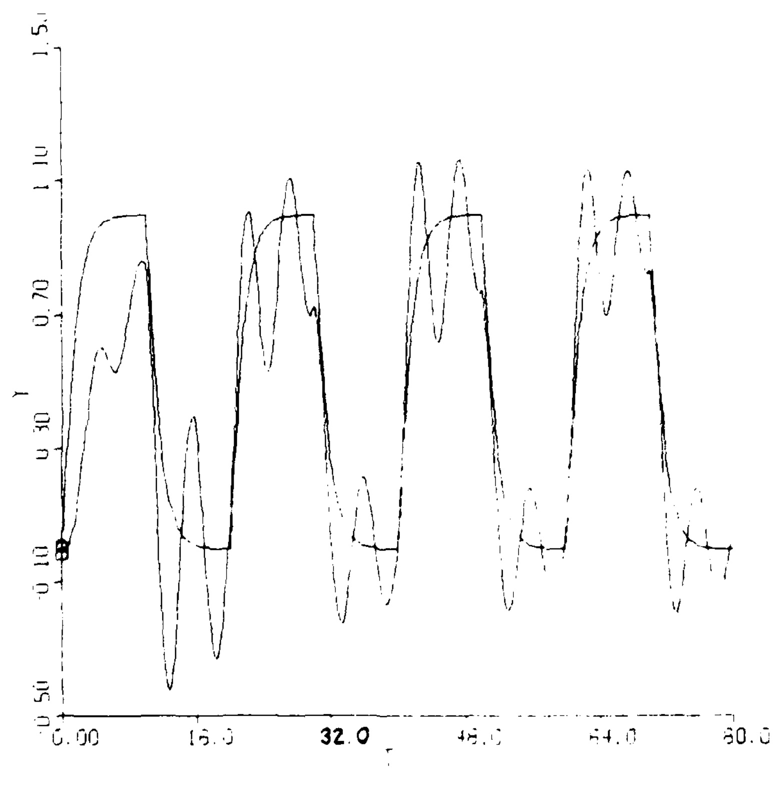


Figure 6.4. MRAC Controller Performance, T,T̄ = 0.1

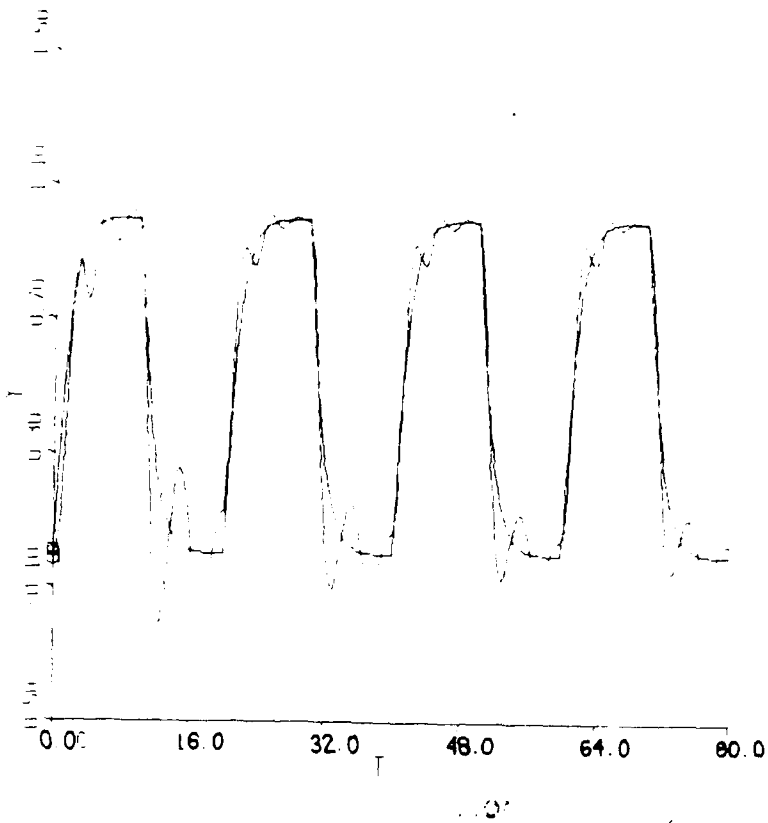


Figure 6.5. MRAC Controller Performance, T,$\bar{T}$ = 1.0

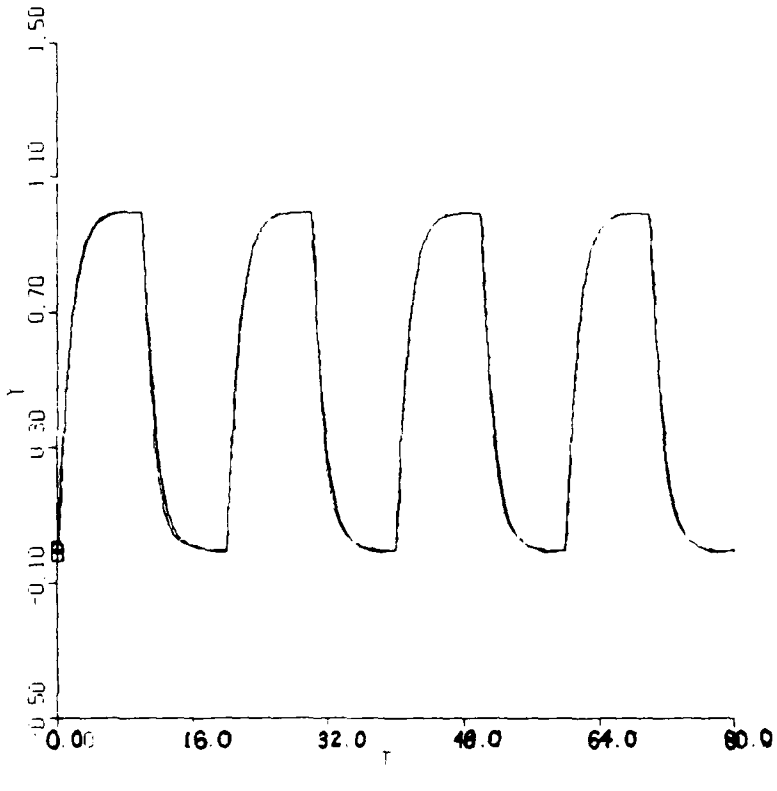


Figure 6.6. MRAC Controller Performance, T,$\bar{T}$ = 10.0

significantly from the expected value. In this case $a_2 = 0.0$ represents a system with no damping even though the controller was designed expecting an overdamped response.

It is immediately obvious as to the impact that the choice of T and $\bar{T}$ has upon the MRAC performance capabilities. The Figures of 6.3, 6.4, 6.5, and 6.6 illustrate the effect of increasing these tuning matrices until the actual plant output is made to coincide exactly with the ideal response.

7.0 Classical Controller Design Comparison

In addition to comparing the adaptive control designs to one another, it is also important to compare the designs with existing accepted technology. This is really the only way to judge the relative merits and limitations of these advanced techniques. In this particular case a traditional Proportional-Integral-Derivative (PID) compensator was chosen for comparison purposes.

A PID controller was placed in series with the nominal plant of the benchmark problem and was tuned to give the desired closed loop response. Figure 7.1 shows both the open loop response (no PID in the loop) and the closed loop response (with PID in the loop) for the nominal plant.

A common form of the PID control law is

$$u = k_p + \frac{k_i}{s} + k_d s$$

In this problem the phase advance supplied by the derivative term was not necessary and was even undesirable, therefore the gain k_d was set to zero. This in effect makes the control law proportional-plus-integral (PI). The gains k_i and k_p were subsequently chosen to provide the closed loop response shown in Figure 7.1. The resulting gains values were

$$k_p = 1.0$$

$$k_i = 0.55$$

$$k_d = 0.0$$

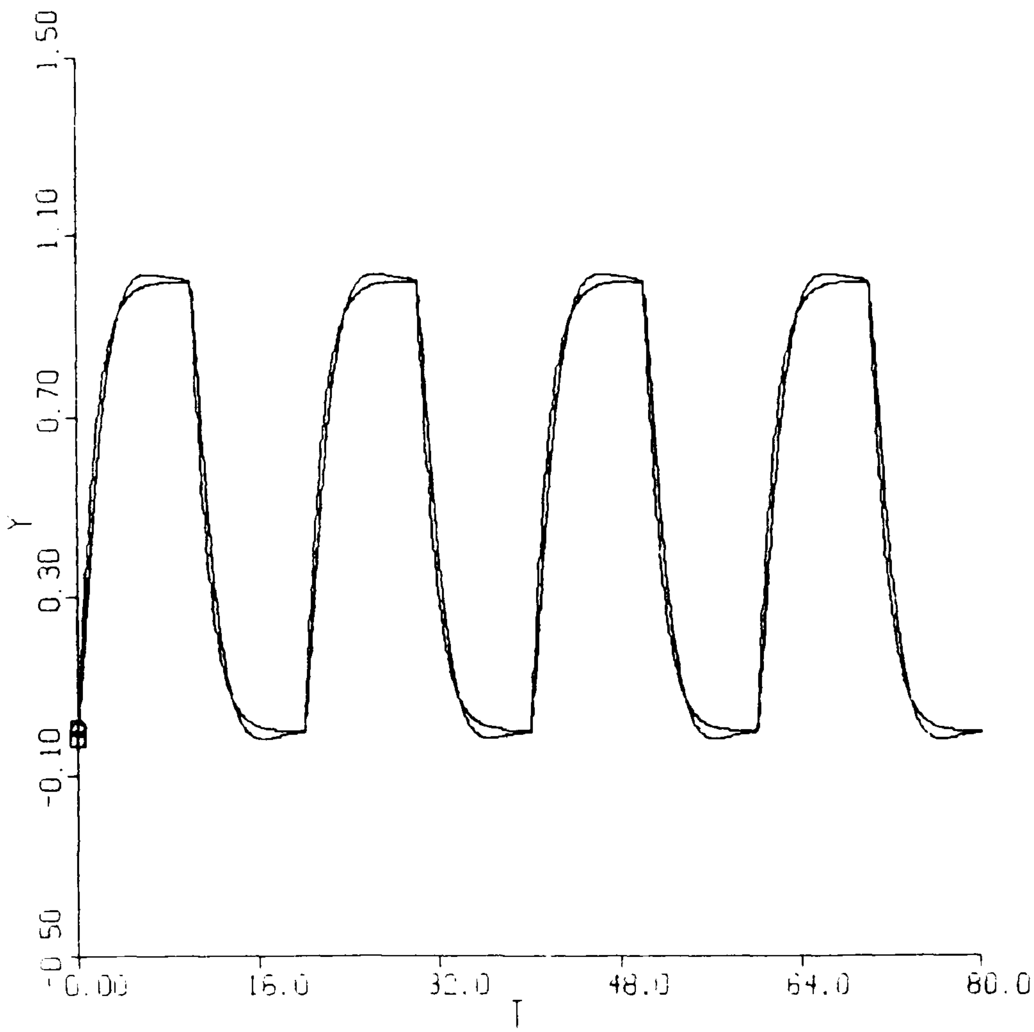


Figure 7.1. PID Closed Loop Response

8.0 Performance Results

The controller designs discussed in sections 4.0 through 7.0 were compared in terms of their performance robustness capabilities when subjected to significant uncertainties due to parameter perturbations, external disturbances, and unmodeled dynamics. The dynamical model used in each of the designs was the plant described in the benchmark problem of section 2.2.

In each case the controllers were judged in terms of their ability to maintain the same desired ideal plant response that was given by an ideal model of the closed loop system. The plant dynamics are governed by

$$\frac{y}{u} = \frac{ks + b}{s^2 + a_2 s + a_1}$$

The ideal closed loop response is given by the plant transfer function above with the following set of nominal parameters

$$\frac{y_{ideal}}{u_{ideal}} = \frac{0.5s + 1}{s^2 + 2s + 1}$$

Figure 8.1 shows the ideal input and desired ideal output for the system.

A performance index was constructed to quantitatively measure the ability of the controllers to maintain the ideal response when subjected to uncertainties and off nominal plant parameters. The performance index was chosen to reflect the type of requirements illustrated in Figure 3.1 of section 3.0. In order to measure both the transient behavior and the steady-state tracking capabilities of the different controllers, the following performance index was used

$$J = \left(\int_0^\infty e^2\, dt\right) \cup (|e|)$$

where

$$e = y_{ideal} - y_{actual}$$

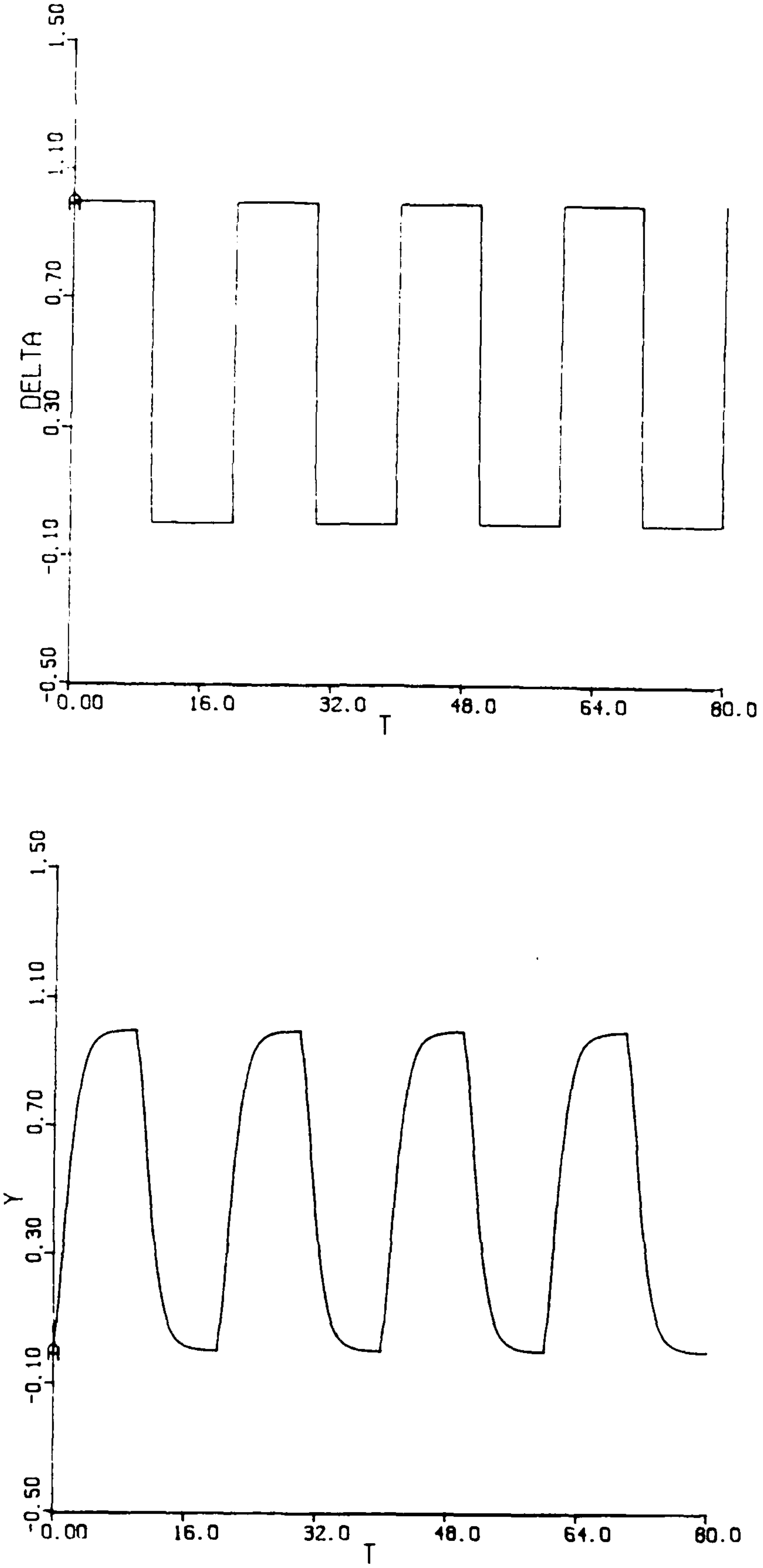


Figure 8.1. Ideal Input and Ideal Output for Closed Loop System

This performance index is the union of Integral-squared-error (ISE) and absolute error (AE), where the AE term penalizes the transient behavior in terms of peak deviation away from the ideal response and the ISE term penalizes poor steady-state tracking performance.

8.1 Parameter Perturbations

The ADAC controller, conventional DAC (polynomial spline), MRAC controller, and PID controller were evaluated in terms of their ability to maintain the ideal model response when subject to parameter perturbations away from the expected nominal values. Each of the controller designs were outlined in sections 4.1, 4.2, 6.0, and 7.0 respectively. Appendix A contains listings of the simulation programs used to perform these comparisons. In the listings are the detailed equations for each of the control laws used. These simulations are written in ACSL (Advanced Continuous Simulation Language) and the equations are self documenting. Figure 8.2 shows a block diagram of the test setup used to evaluate the controller designs.

In each case the controller was placed in the closed loop with the nominal plant. The plant parameters were then varied away from their nominal values and the performance index was monitored. The parameters were individually varied until the performance index exceeded a prescribed threshold. By individually varying the parameters the entire parameter space was explored and certain areas of suitable performance were established. The threshold on the ISE term of the performance index was 1.0 and the threshold for the AE term was 0.2. As long as these thresholds were not exceeded the controller was able to maintain the desired closed loop response satisfactorily.

Figures 8.3, 8.4, and 8.5 show plots of the ADAC controller, Polynomial Spline DAC, and PID controllers in the a_1 - a_2 parameter space for the fixed values of $b = 1.0$, $b = 3.0$, and $b = 10.0$ respectively. Note, only a_1, a_2, and b were varied in this study, so the parameter space can be visualized as a three dimensional Euclidian space. The effect of varying b was to change the location of the plant zero, which indirectly effects the gain term as well, therefore to simplify the results the gain parameter k was maintained at the nominal value of 0.5 always. Keep in mind that the enclosed regions in Figures 8.3, 8.4, and 8.5 depict areas of unexpected off-nominal parameter variations where the ideal model response was maintained. This distinction is important in that these regions indicate the performance robustness of the various controller designs, as opposed to the more popular but less important stability robustness which is often cited in controls literature.

Examination of Figures 8.3, 8.4, and 8.5 reveals that both of the DAC controllers are far superior to the PID controller, as was expected. Variations in terms of orders of magnitudes were tolerable for the ADAC and Polynomial

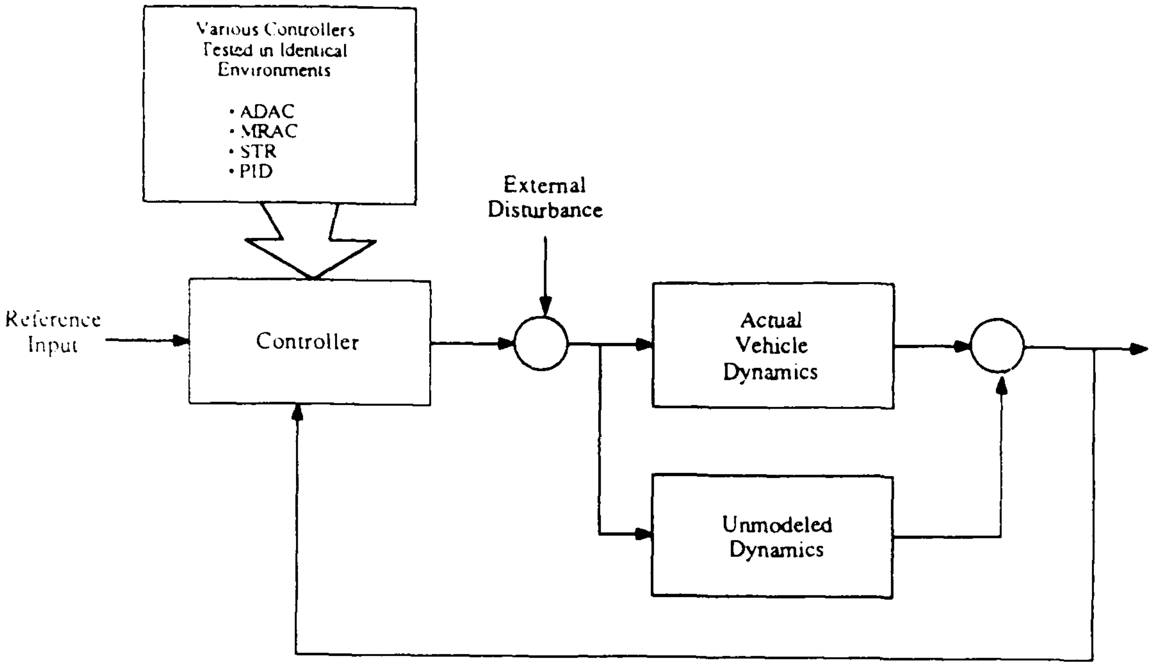


Figure 8.2. Controller Test Block Diagram

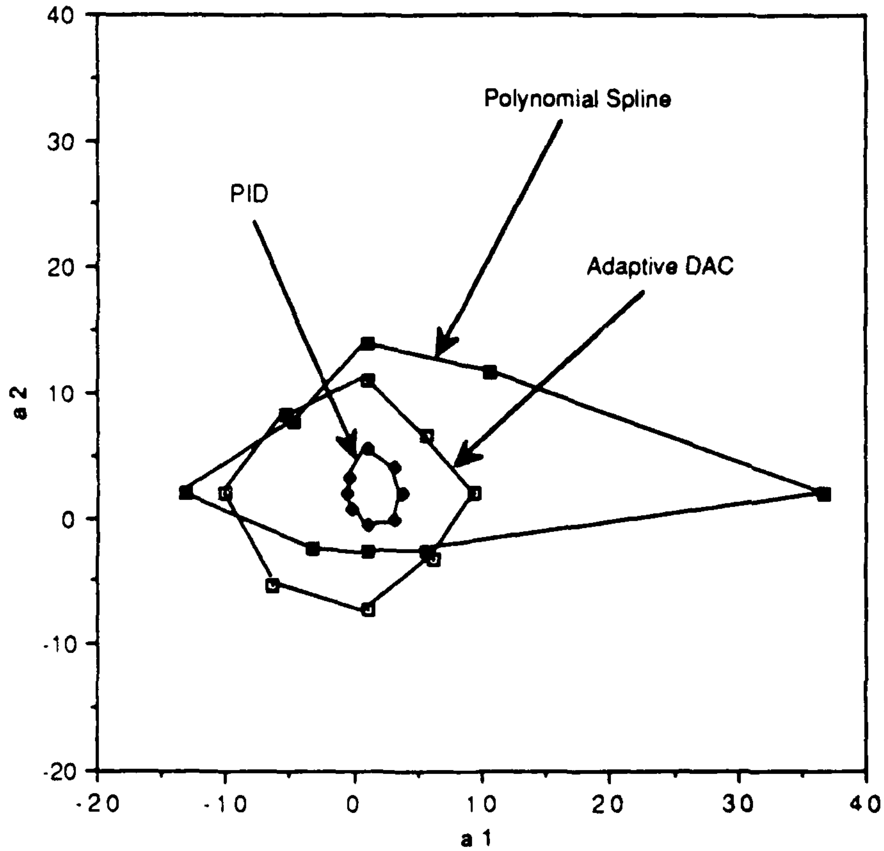


Figure 8.3. Parameter Space Performance Plots, b=1.0

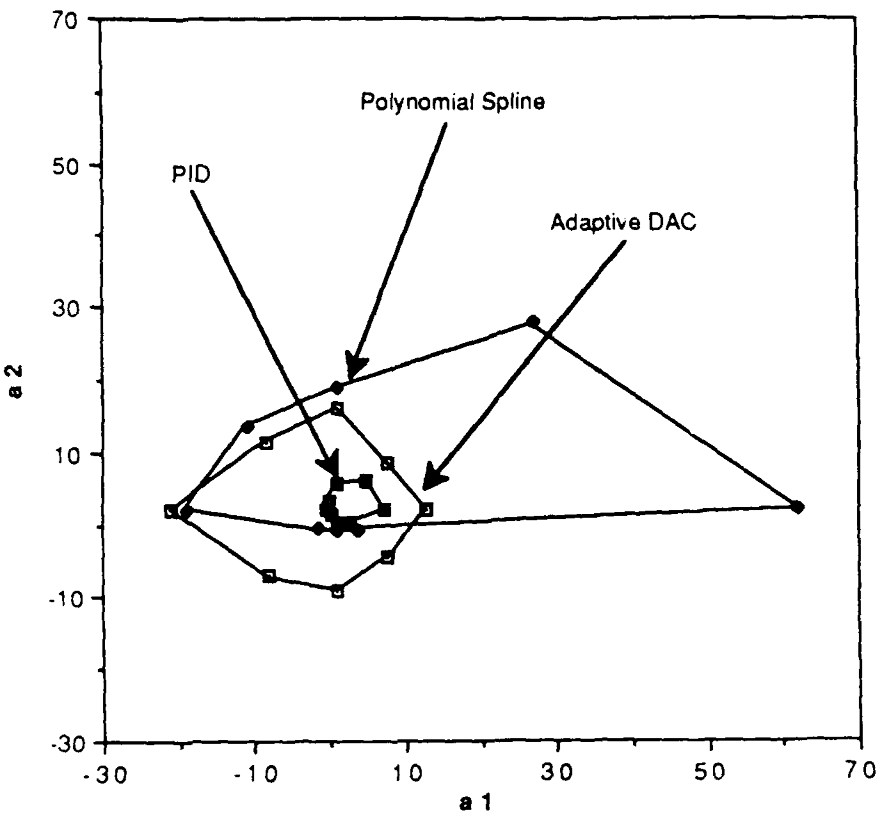


Figure 8.4. Parameter Space Performance Plots, b=3.0

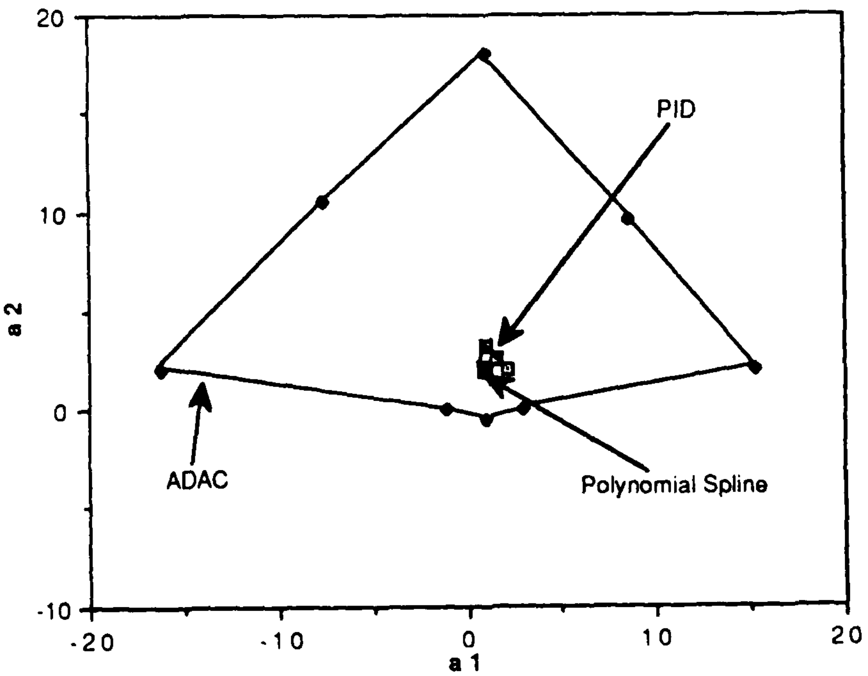


Figure 8.5. Parameter Space Performance Plots, b=10.

Spline DAC controllers. Notice in Figures 8.3 and 8.4 that the polynomial spline DAC seems to be relatively insensitive to variations in a_1, while the ADAC design reflects a nice symmetry about the nominal plant parameters. This observation is even more apparent in Figures 8.6 and 8.7 which show the ADAC and Polynomial Spline DAC controllers separately, for the different values of b. However, in both Figure 8.5 and 8.7 it is evident that the Polynomial Spline DAC is very sensitive to variations in b, and for b = 10.0 the region of acceptable performance has collapsed to a single point about the nominal plant values. The ADAC design does not reflect such a sensitivity to b variations, and Figure 8.6 indicates that even for the case b = 10.0 the area of acceptable a_1 - a_2 variations is still quite large. This set of plots indicates that if the regions of acceptable performance were stacked on top of each other in a three dimensional (a_1, a_2, b) space, that the resulting volume of acceptable performance for the ADAC controller would be larger than the corresponding volume for the Polynomial Spline DAC controller. If only the plots of Figures 8.3 and 8.4 were considered, it would appear that the polynomial spline DAC controller was better than the ADAC controller, but if the entire volume of parameter variations is considered the ADAC controller is better. The decreased insensitivity of the polynomial spline DAC compared to the ADAC for certain parameter variations shown in Figures 8.3 and 8.4 is due to the increased ability of the polynomial spline model to estimate particular characteristics of the system uncertainty as compared to the exponential basis functions used in the ADAC controller. Figure 8.8 shows the PID controller performance for the same set of b variations.

The results of Figures 8.6, 8.7, and 8.8 are somewhat misleading as they indicate the controllers better performance for off nominal values of b. This is an artifact of the choice of performance index. For the value of b = 3.0 the performance region is larger than that for b = 1.0 or b = 10.0 since neither the absolute error (AE) <u>or</u> the integral-squared-error (ISE) terms in the performance index dominates. Rather, both terms are relatively equal in magnitude. For the cases of b = 1.0 and b = 10.0, either the ISE or AE term dominates the performance index and the boundary is defined by primarily one term or the other. The particular combination of parameter variations for the case b = 3.0 results in the boundary being defined by a combination of both terms equally. Another choice of performance index (strictly ISE alone or AE alone would not show the performance improving as b moved away from nominal). The performance index chosen is still valid in terms of making comparisons, however, since all controllers were judged using the same criteria.

The dynamical order of the ADAC and Polynomial Spline DAC controllers was the same (6th order). This is in contrast to the PID controller which has a dynamical order of one. The poles of the composite observers for both the ADAC and Polynomial Spline designs were identically located at (-3.0) in the complex plane (all poles).

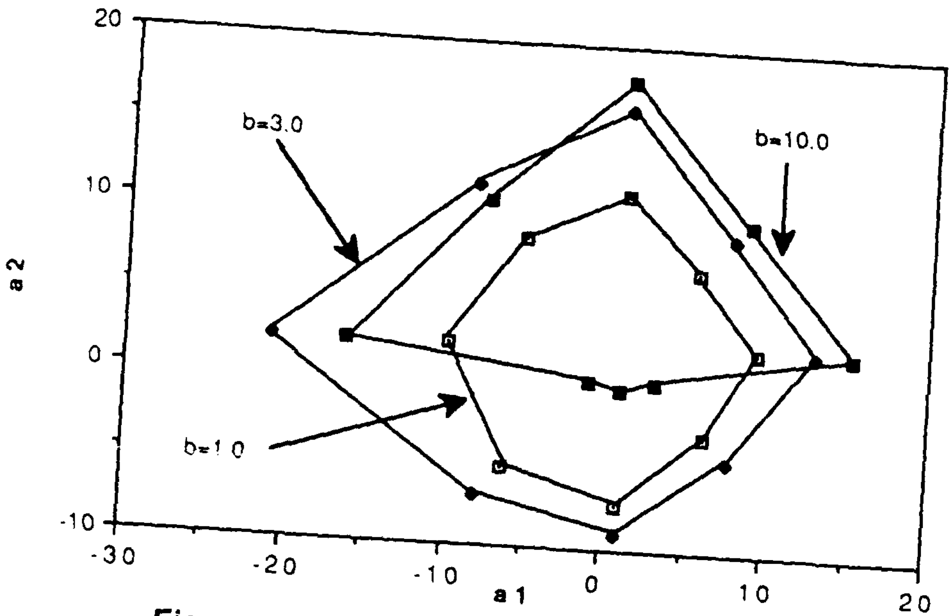


Figure 8.6. ADAC Controller Performance

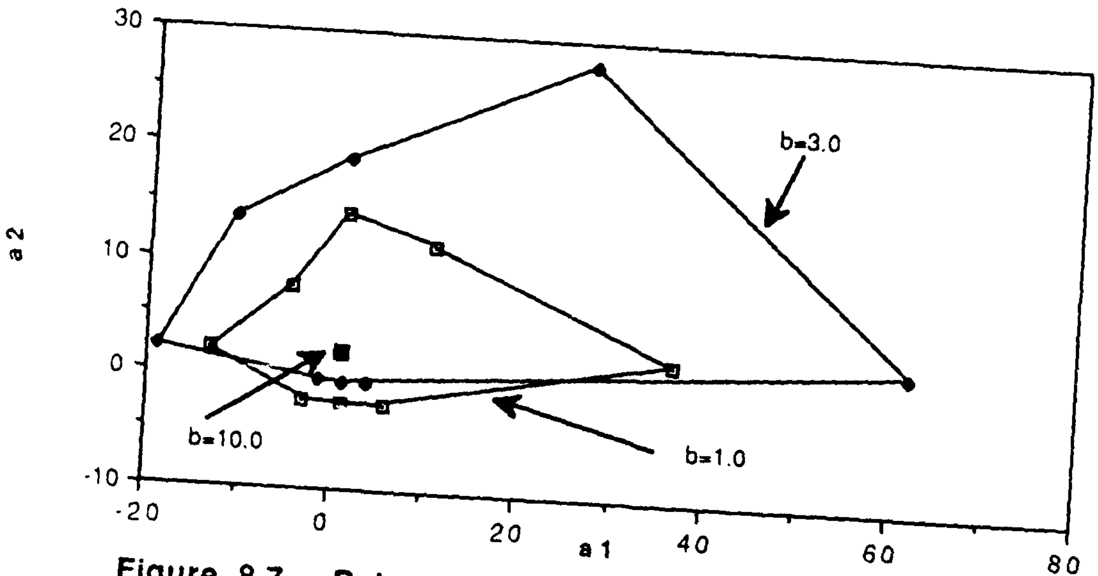


Figure 8.7. Polynomial Spline DAC Controller Performance

Figure 8.9 shows a similar set of plots for the MRAC controller design. The performance area are plotted for several values of the tuning matrices T and $\bar{T}$. It is obvious that these tuning parameters could be increased until the regions of acceptable performance covered practically the entire parameter space. For this reason it is very difficult to compare the the results of the DAC controllers with the MRAC, since we have no way of gauging a set of realistic values for the tuning matrices T and $\bar{T}$.

A set of typical transient responses and the associated control signals for the ADAC, MRAC, and PID controllers are shown in Figures 8.10, 8.11 and 8.12 respectively. These responses are indicative of the types of responses existing when the system is very close to one of the boundaries shown in the parameter space plots above. In other words, these transient responses represent "worst-case" acceptable responses. For the ADAC controller the particular parameter values were $b = 1.0$, $a_1 = -5.0$, $a_2 = -5.0$. For the MRAC controller these parameters were $b=1.0$, $a_1 = 10.0$, $a_2 = -5.0$, $T = 10.0$. For the PID controller these parameters were $b=1.0$, $a_1 = 1.0$, $a_2 = -0.5$.

8.2 External Disturbances

The baseline closed loop system was subjected to external disturbances as shown in Figure 8.13.

Each of the adaptive controller designs ADAC, MRAC, Polynomial Spline DAC, and the classical PID design use some type of integral feedback in their control laws. This means that each of these controllers will be capable of some degree of disturbance rejection. The actual degree of rejection possible is a function of the controller's response time (or inversely, the controller bandwidth).

The response time of the DAC controllers is set by the choice of the observer poles. The deeper the poles are placed into the left-half of the complex plane, the faster the response, and the greater the bandwidth. For the MRAC controller, the response time is set by the choice of tuning matrices T and $\bar{T}$. The greater the value of T and $\bar{T}$, the faster the response, and the greater the bandwidth. For the PID controller the disturbance response is set by the choice of the integral gain k_i.

To demonstrate the disturbance rejection capabilities of each controller, they were individually subjected to a set of disturbances that covered the entire possible spectral space of the controller's bandwidth. This disturbance set was comprised of two basic signals. The first disturbance signal was a single sinusoid, varied in frequency from dc to beyond the controller bandwidth. This single spectral line swept through the entire controller

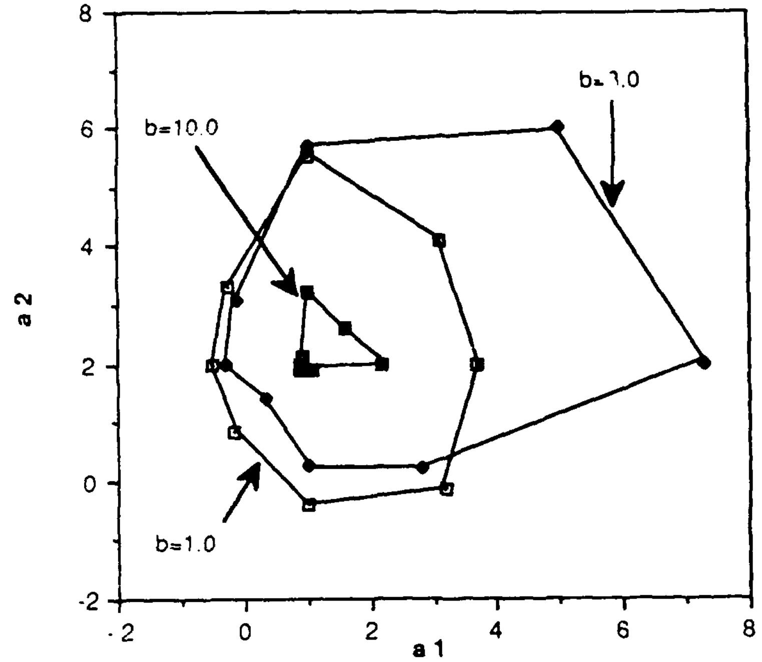


Figure 8.8. PID Controller Performance

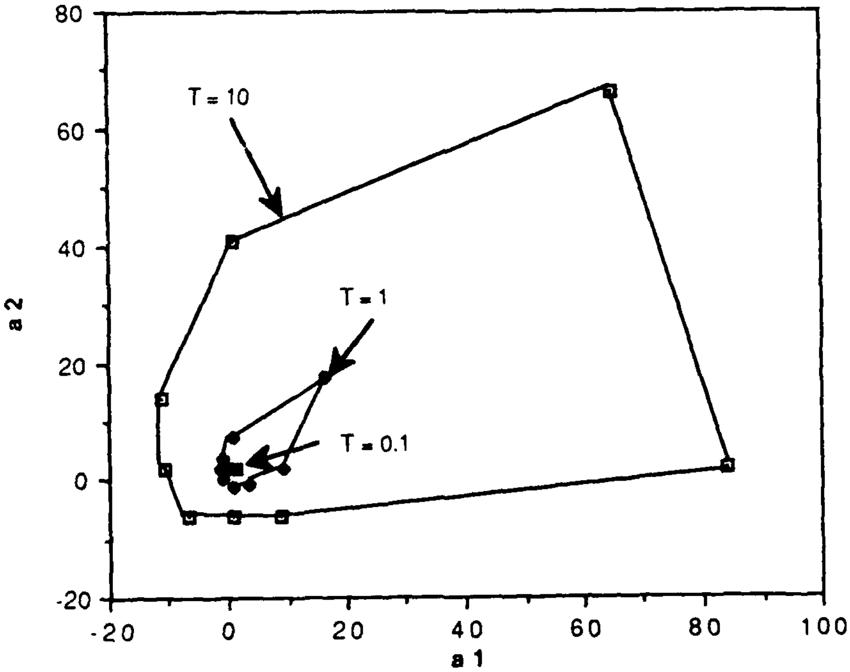


Figure 8.9. MRAC Controller Performance

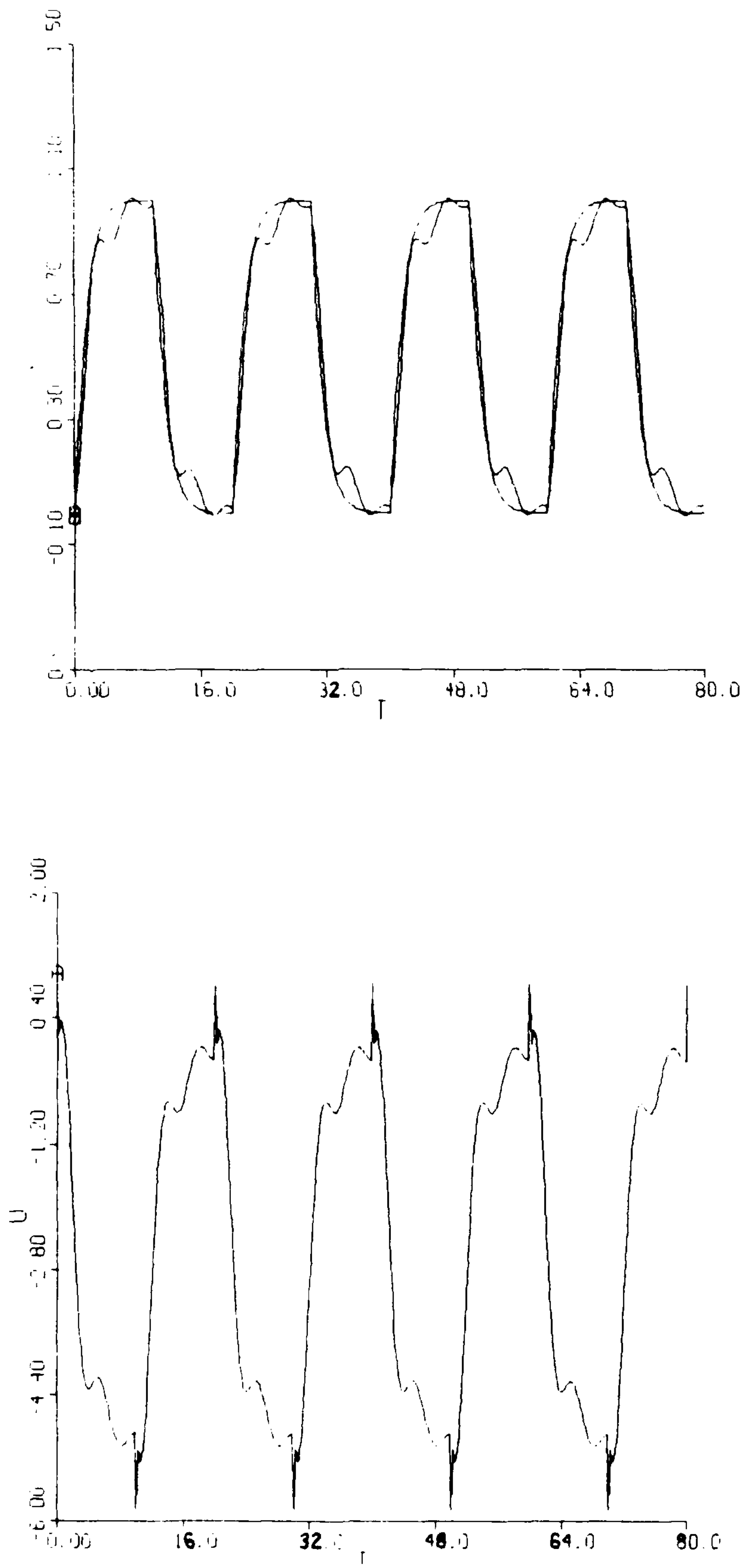


Figure 8.10. ADAC Controller Transient Response and Control Signal

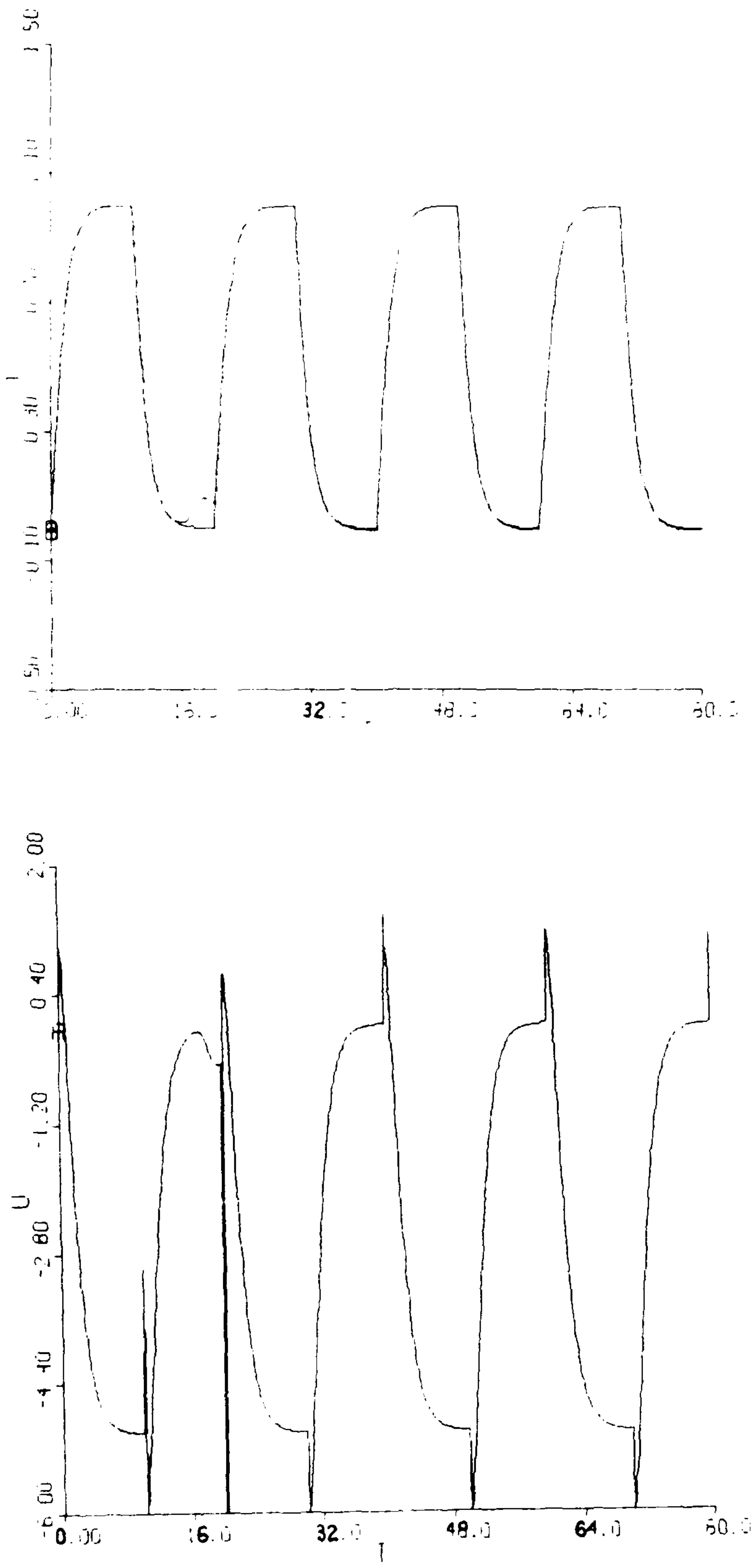


Figure 8.11. MRAC Controller Transient Response and Control Signal

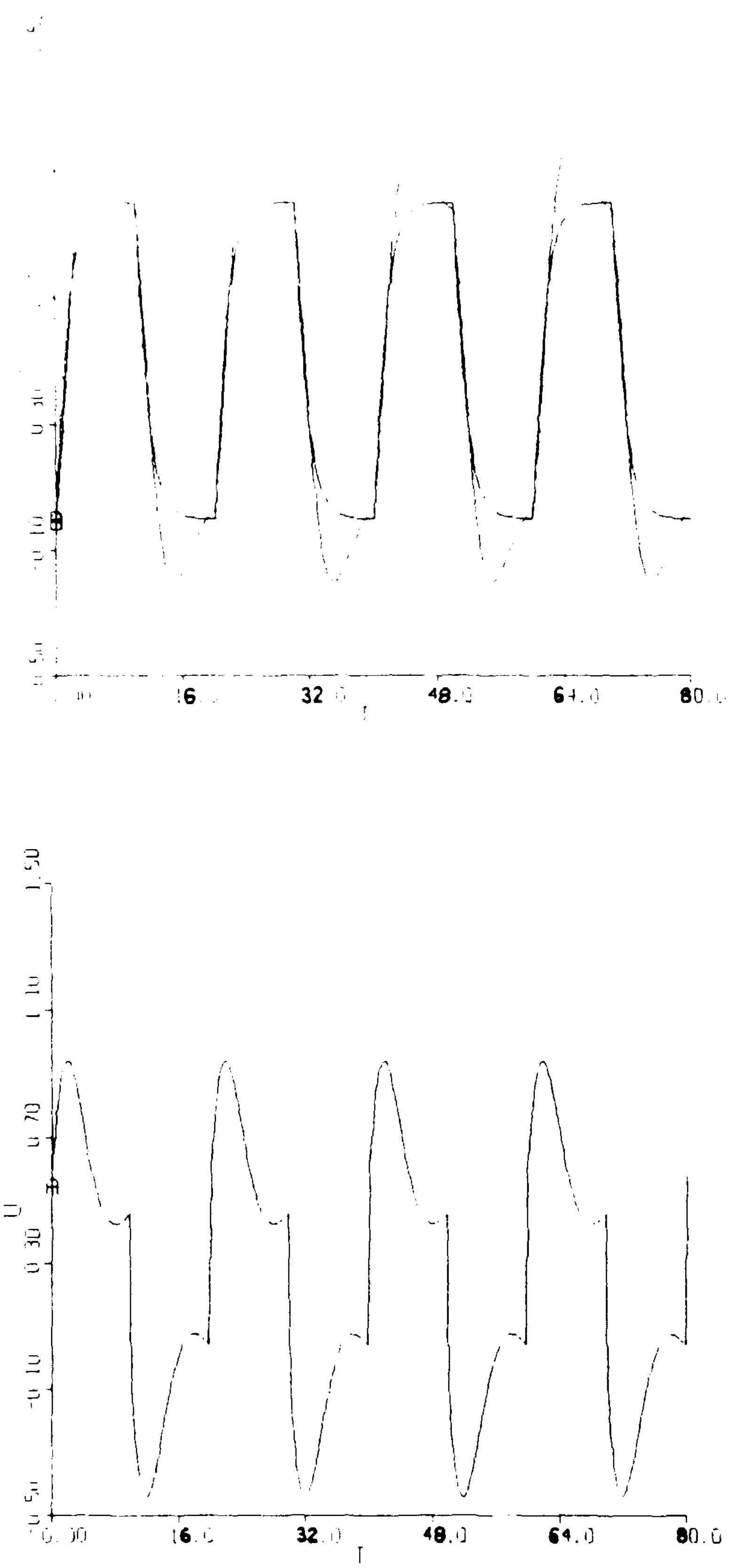


Figure 8.12. PID Controller Transient Response and Control Signal

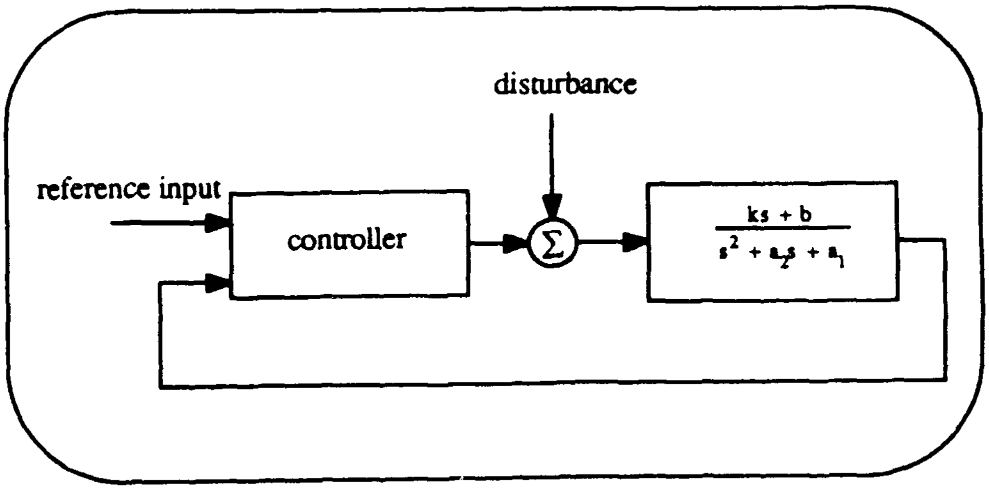


Figure 8.13. External Disturbance Inputs to the System

spectrum represents every single disturbance frequency possible. The second disturbance signal was wideband white noise. The spectral content of white noise is a continuum of frequencies which cover the controller bandwidth. This choice of disturbance represents every possible combination of disturbance frequencies. These spectral characteristics are shown graphically in Figure 8.14. Time domain examples of these disturbance signals are shown in Figures 8.15 and 8.16.

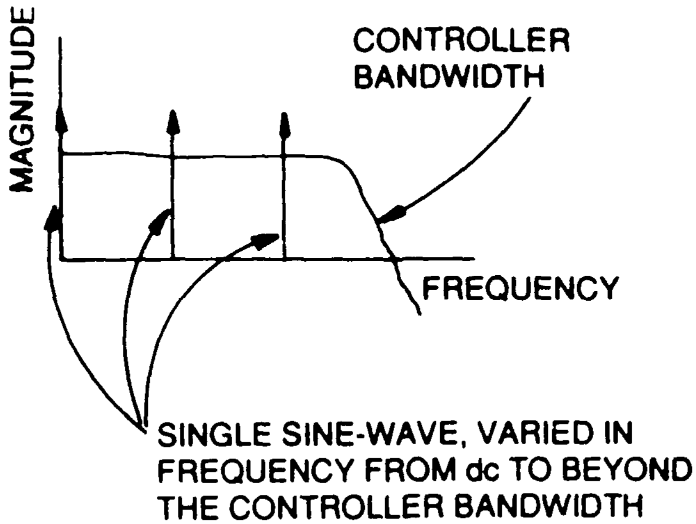
The response of the ADAC, MRAC, and PID controllers to these disturbances are shown in Figures 8.17, 8.18, and 8.19 respectively. Figure 8.17 illustrates the ability of the ADAC controller to maintain the ideal response characteristics in the presence of any disturbance which has a spectral content that is within the controller bandwidth. The disturbance rejection capabilities of the ADAC controller are not surprising since the entire concept of DAC control was originated for this particular purpose. The disturbance rejection performance can be better appreciated by comparing Figure 8.17 with Figure 8.19 which shows the PID controller performance. The PID has limited capability due to the single integrator available in the controller. In fact it can be shown analytically that the only type of disturbance which can be completely canceled by this PID controller is a constant disturbance.

The actual outputs (control signals) of the ADAC, MRAC, and PID controllers are shown in Figures 8.20, 8.21, and 8.22 for comparison purposes. These are the control signals for the case of the single sinusoidal disturbance.

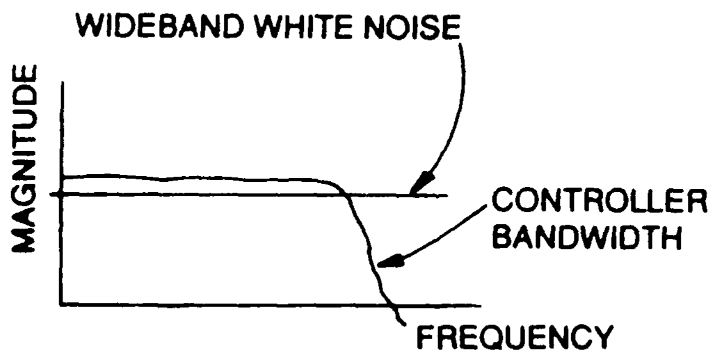
8.3 Unmodeled Dynamics

Unmodeled dynamics were introduced into the baseline system. The unmodeled dynamics can be separated into two categories: (1) multiplicative and (2) additive. The distinction between these two types of unmodeled dynamics has a significant impact on the control system design and performance.

Consider the case of multiplicative unmodeled dynamics shown in Figure 8.23. This Figure shows the characteristic pole locations in the complex frequency plane for a system with eight total poles, two of which are modeled in the assumed dynamics. Those poles lying well to the left of the modeled poles are assumed to be "insignificant" in terms of their effect on the overall system. The fact that these poles do not contribute significantly to the response of the system allows them often to be disregarded during the design process. This is standard practice in control system design and is a well-accepted and valid approach. There are four other unmodeled poles in this example which lie to the right of the modeled poles. The fact that they lie to the right in the complex plane means that these poles will contribute



(1) EVERY SINGLE POSSIBLE FREQUENCY



(2) EVERY POSSIBLE COMBINATION OF FREQUENCIES

Figure 8.14. Spectral Characteristics of External Disturbances

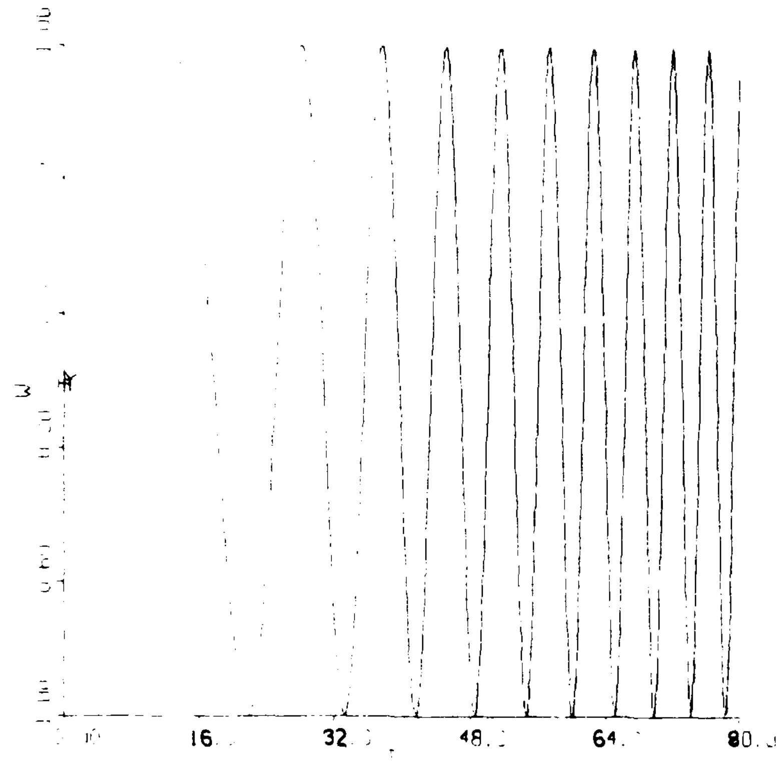


Figure 8.15. Transient Plot of Single Sinusoidal Disturbance

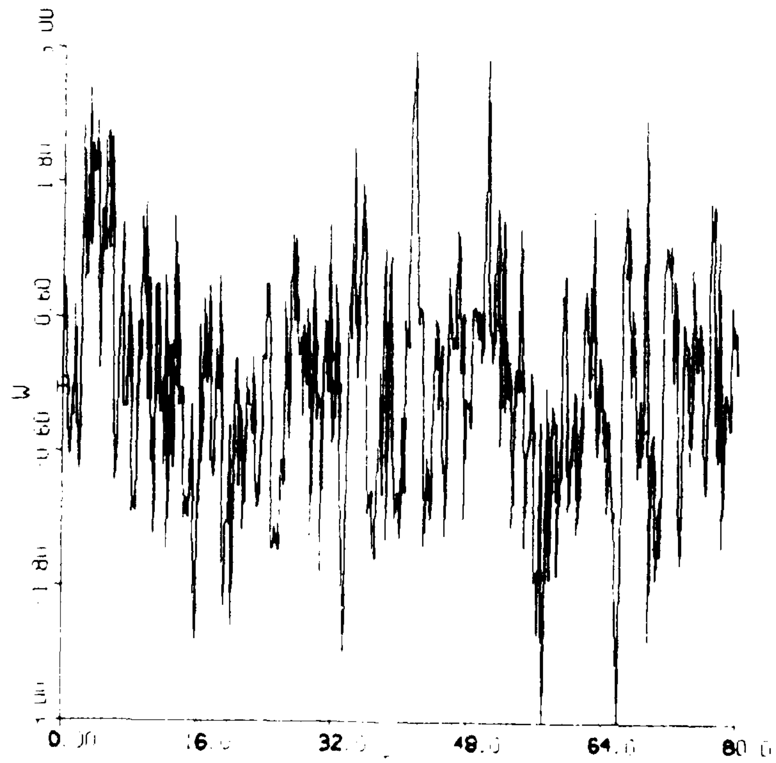
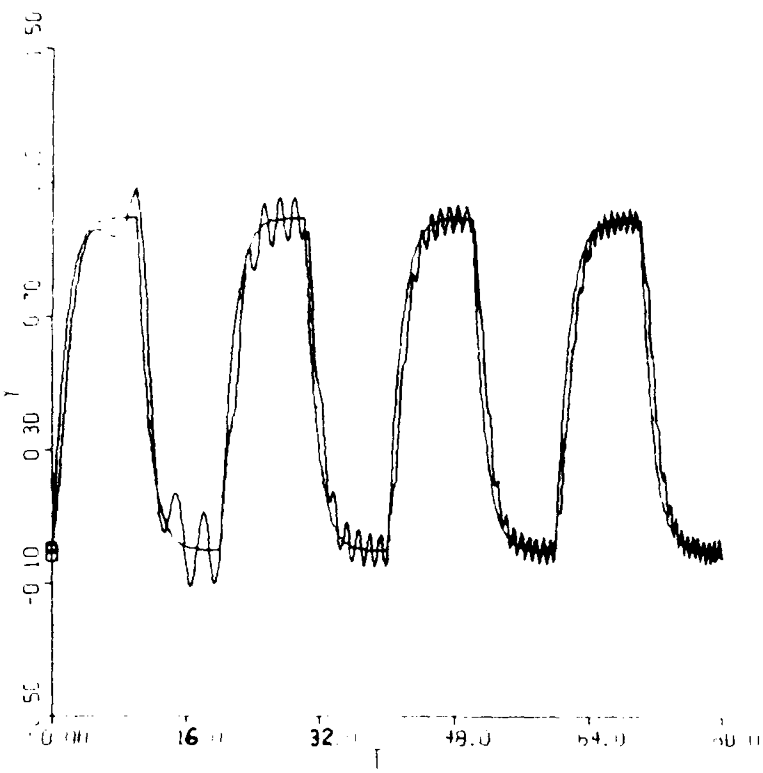
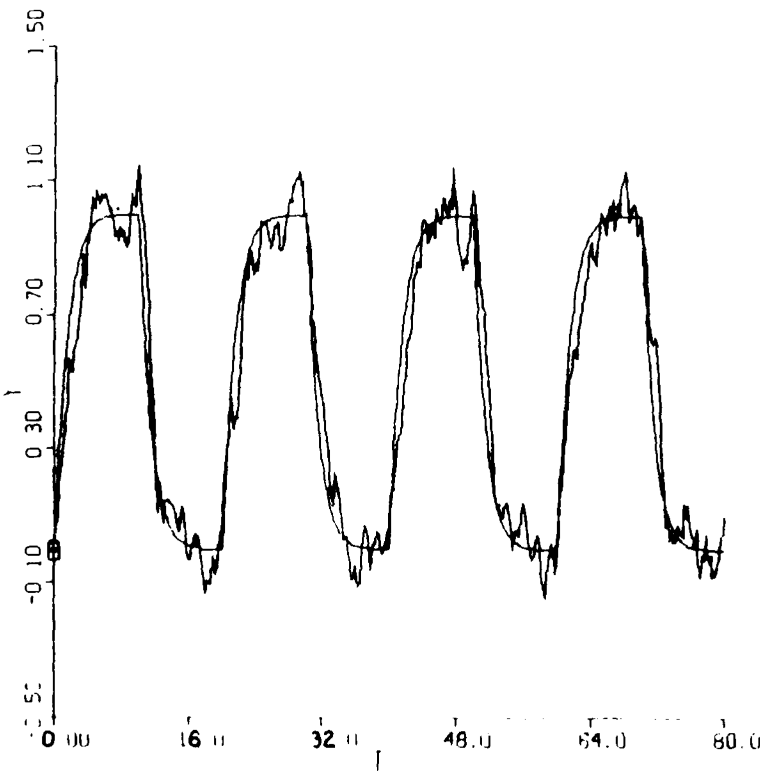


Figure 8.16. Transient Plot of Wide Band Noise Disturbance

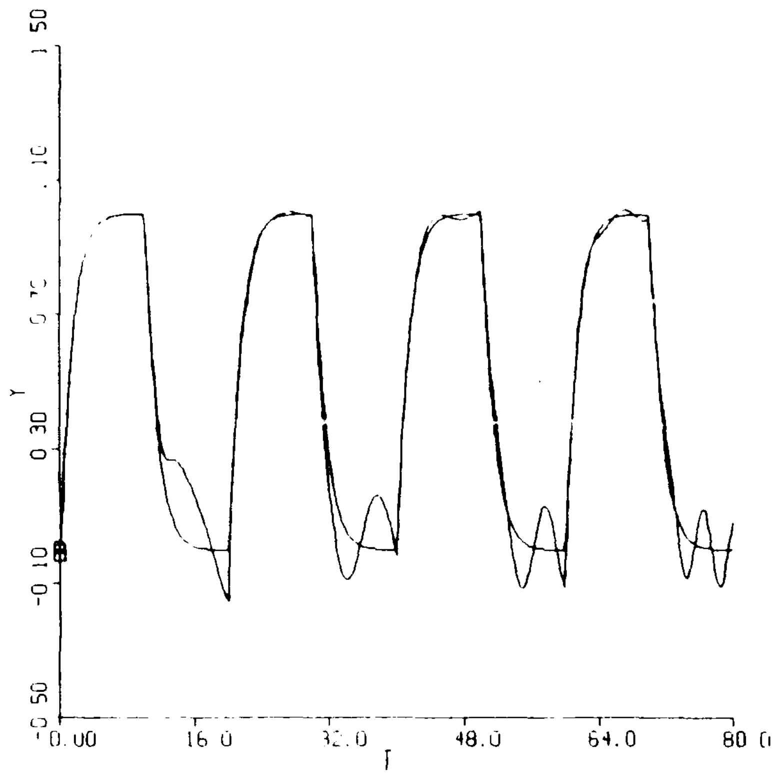


• Single sine wave input, frequency swept from 0 - 8 rad/sec

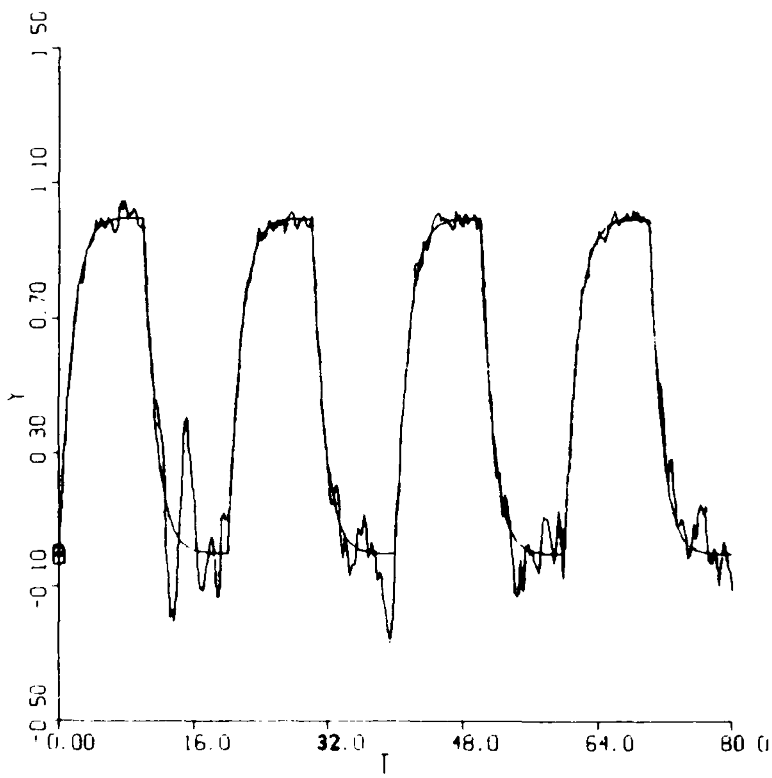


• Wide band white noise input

Figure 8.17. ADAC Controller Response to External Disturbances

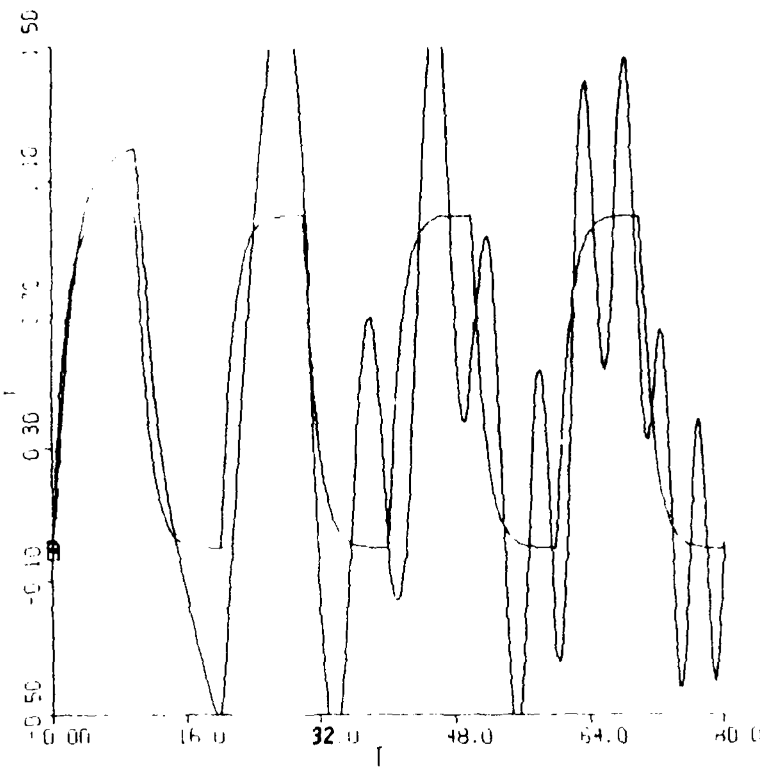


• Single sine wave input, frequency swept from 0 - 8 rad/sec

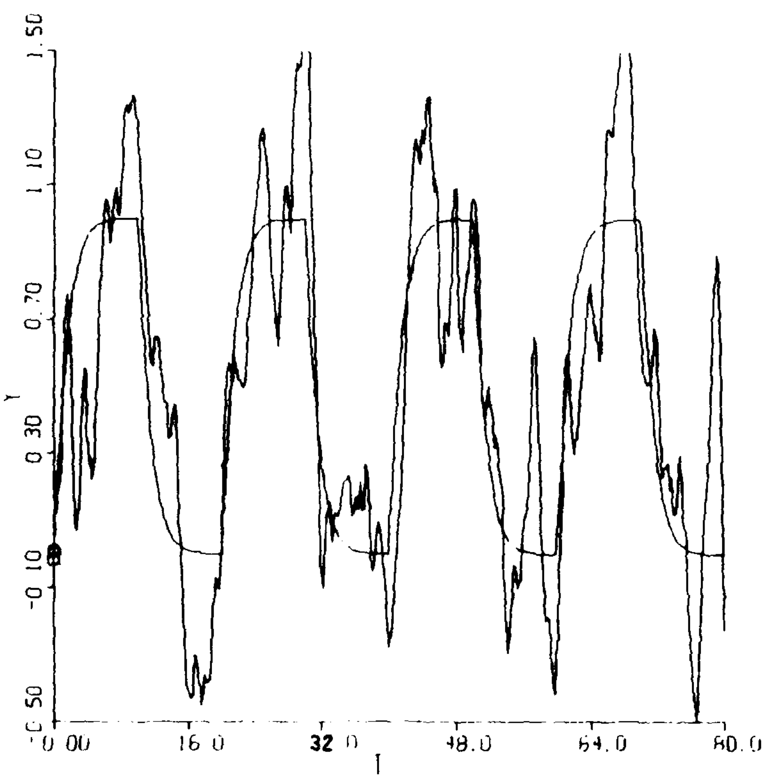


• Wide band white noise input

Figure 8.18. MRAC Controller Response to External Disturbances



• Single sine wave input, frequency swept from 0 - 8 rad/sec



• Wide band white noise input

Figure 8.19. PID Controller Response to External Disturbances

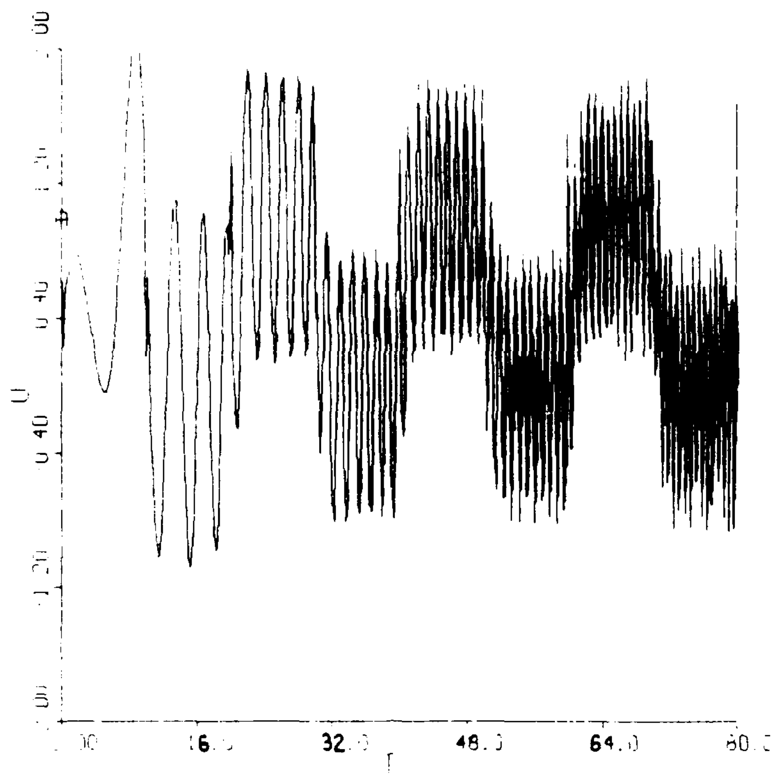


Figure 8.20. ADAC Control Signal, Sinusoidal Disturbance Input

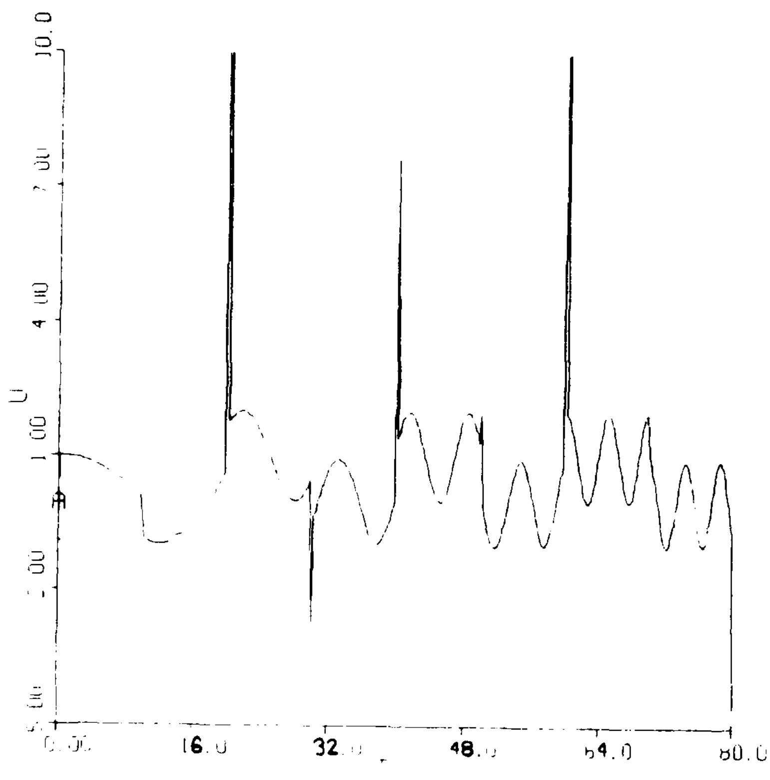


Figure 8.21. MRAC Control Signal, Sinusoidal Disturbance Input

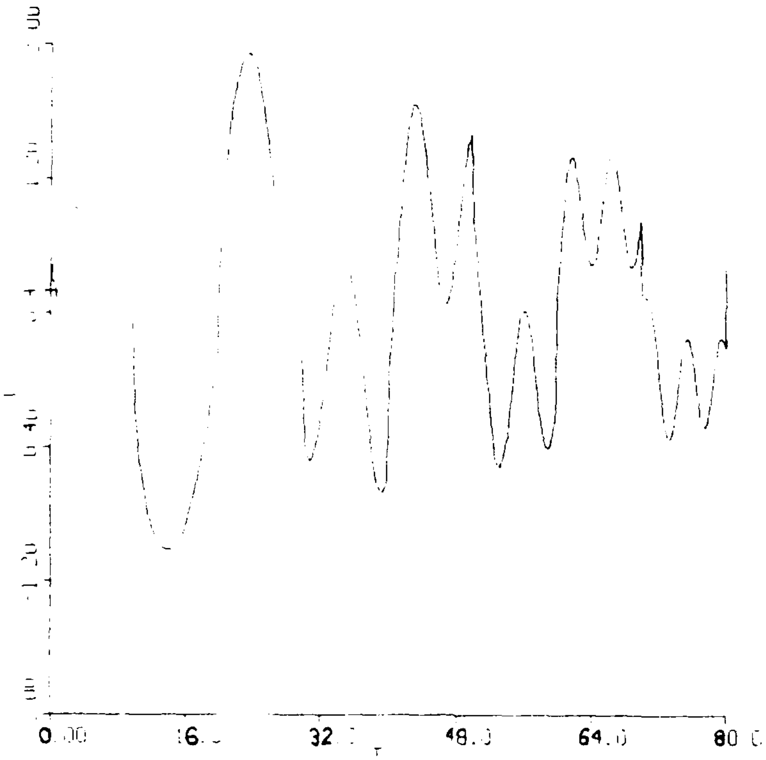


Figure 8.22. PID Control Signal, Sinusoidal Disturbance Input

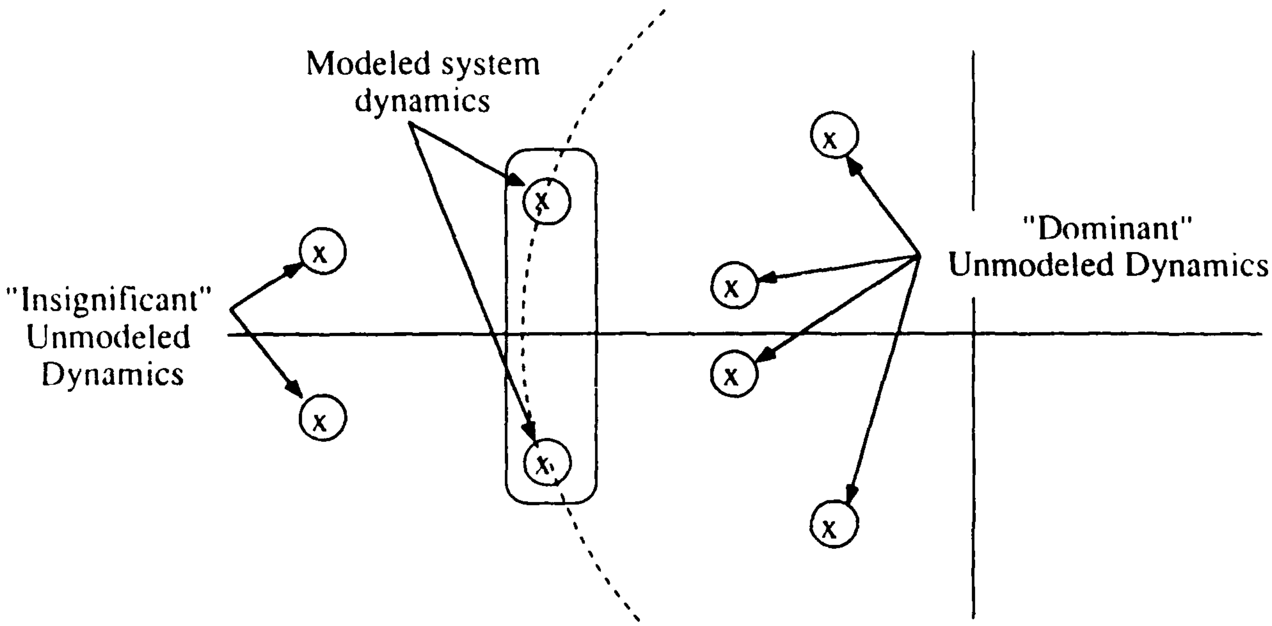


Figure 8.23. Characteristic Poles of Multiplicitive Unmodeled Dynamics

significantly to the overall response and, in fact, will dominate the system response. Designing a control system based upon particular modeled dynamics while neglecting dominant unmodeled poles is tantamount to designing a controller based upon an invalid reduced-order model (ROM). The problems associated with control system design based upon ROM's is well known and documented [14].

None of the adaptive control designs (Polynomial DAC, ADAC, and MRAC) were able to maintain the performance requirements in the presence of dominant multiplicative unmodeled dynamics. This really is a somewhat impractical problem in that it is desired to design a control system for a plant in which the dominant characteristics are unknown, and an invalid ROM is used instead. Not even adaptive control offers a solution to this type of problem.

The case of additive unmodeled dynamics is a completely different situation. Additive unmodeled dynamics act as effective external disturbances on the closed loop system. It is apparent that the controller's performance should be similar to that outlined in section 8.2. This is in fact the case and Figures 8.24 and 8.25 show an example of this performance for the ADAC controller, when a pair of undamped unmodeled poles are added to the closed loop system as shown in Figure 8.26 (the frequency of these undamped poles was 0.25 cycles per second). Figure 8.24 illustrates the response in the open loop case (no ADAC control), and Figure 8.25 shows the closed loop performance (with ADAC control). The ADAC system performs very well in the presence of this particular additive unmodeled dynamical uncertainty.

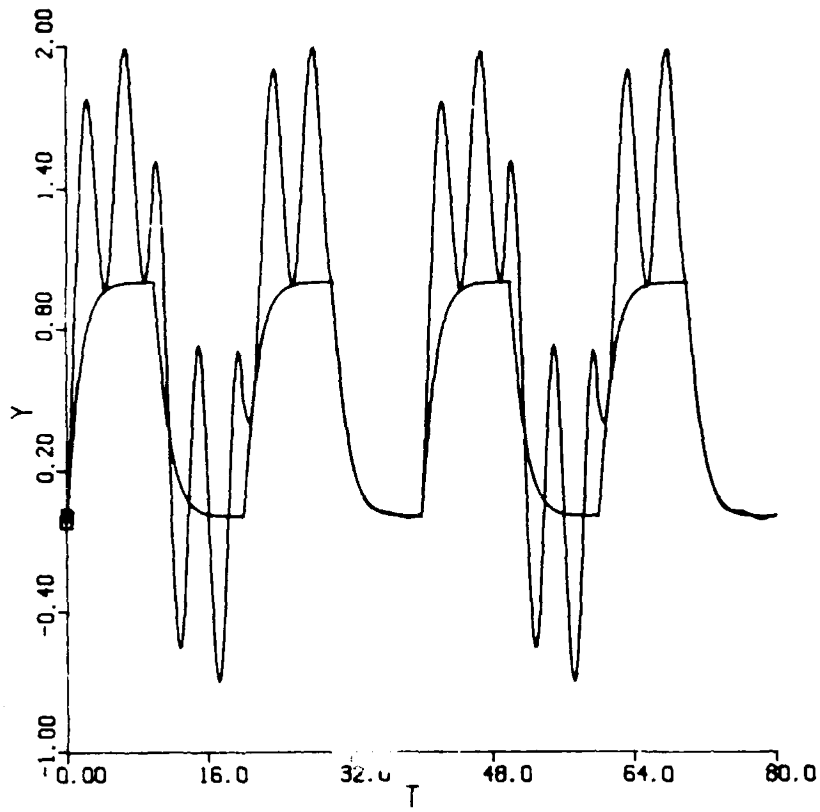


Figure 8.24. Additive Unmodeled Dynamics, Open Loop Response

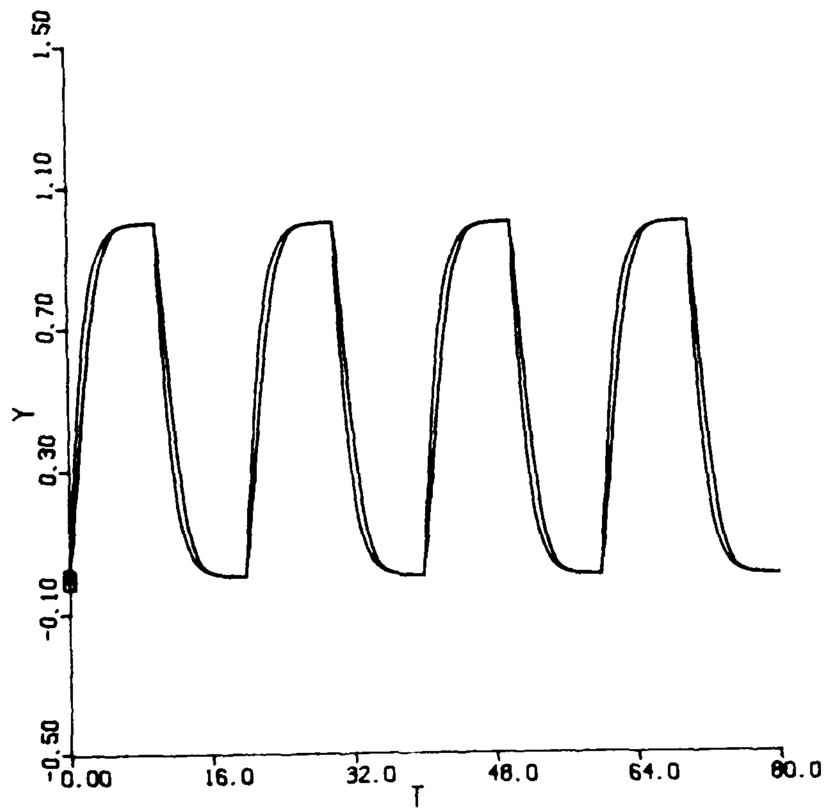


Figure 8.25. Additive Unmodeled Dynamics, ADAC Closed Loop Response

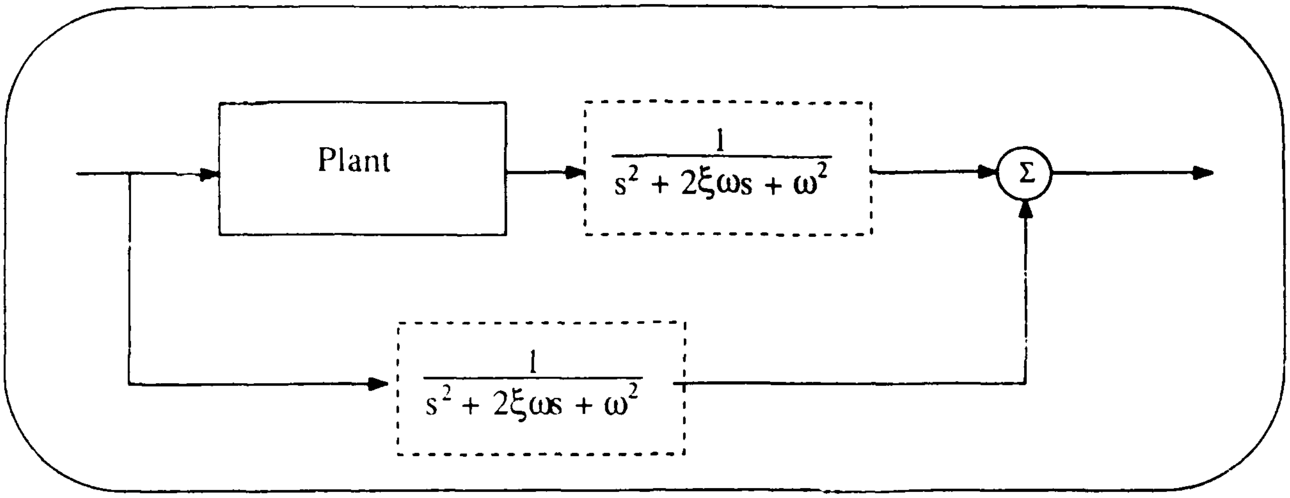


Figure 8.26. Transfer Function Format of Unmodeled Dynamics

9.0 Summary

Disturbance Accommodating Control principles were applied to the design of a baseline control system. The resulting DAC controller designs were evaluated in terms of their ability to maintain an ideal model response when the closed loop system was subjected to significant uncertainties. These uncertainties included parameter perturbations, external disturbances, and unmodeled dynamics. The DAC designs were compared to other adaptive techniques including Self-Tuning Regulator and Model Reference Adaptive Control concepts. These designs were also compared to a classical Proportional-Integral-Derivative controller. All of the adaptive controller designs were judged relative to each other and relative to the PID design, in terms of performance, design complexity, and confidence of operation (reliability).

For the case of parameter perturbations, performance results were calculated and used to determine specific regions in a three dimensional parameter space where the controllers were able to maintain ideal response characteristics. These regions were centered about some nominal set of parameter values, and their size indicates the amount of parameter uncertainty allowable for the various controller designs.

For external disturbances it was shown that the disturbance rejection capability was a function of the bandwidth of the controller and the spectral content of the disturbance. This is the case for each of the controller designs which use some type of integral feedback. Disturbance rejection was verified by subjecting the closed loop system to a class of external disturbances which spanned the complete spectral space possible. This set of external disturbances was composed of two basic components: a single sinusoid varied in frequency to represent every single possible disturbance frequency, and broad band white noise to represent every possible combination of disturbance frequencies.

Unmodeled dynamics were classified into two categories: multiplicative and additive. Additive unmodeled dynamics were shown to have much the same effect as external disturbances and compensated for much in the same manner by the various controllers. Multiplicative unmodeled dynamics were shown to present a problem to all of the adaptive controller designs, in that none of the controllers were able to maintain performance margins in the presence of dominant multiplicative unmodeled poles.

The ADAC controller designs were shown to posses very good performance robustness in the presence of each of the uncertainties discussed. In terms of parameter perturbations the ADAC controller was able to withstand variations of one order of magnitude for both the numerator and denominator coefficients of the nominal plant given in the baseline problem.

This represents a vast increase in capability over the classical PID design, which was expected. It should be reiterated that all of the results shown are in terms of performance, which was defined as the ability of the controller to maintain an ideal model transient and steady-state tracking response. This distinction further magnifies the regions of allowable parameter uncertainty shown in the Figures of section 8.1.

When design complexity and confidence of operation are considered as criteria along with performance robustness, then it becomes very obvious that the DAC based designs are superior to the other adaptive control techniques. The MRAC system exhibited very good performance characteristics for certain choices of tuning parameters, but the inherent nonlinearity of the resulting controller tends to limit the applicability of this method. The only tools available to analyze and measure the reliability and robustness of the closed loop system are repeated simulation and Liapanov stability analysis. The resulting DAC based controllers are linear time-invariant systems, which means that all of the standard linear analysis tools (Nyquist, Bode, root locus, etc.) are still applicable. Again, the STR approach resulted in a nonlinear closed loop system which was subject to the same analysis problems as the MRAC. Even beyond the issue of nonlinearity, however is the fundamental limitation of the estimation of system parameters in the presence of external disturbances. Figure 9.1 shows a matrix which details these comparisons.

	ADAC	MRAC	STR
Performance	Excellent performance characteristics for parameter perturbations, external disturbances, and additive unmodeled dynamics.	Excellent performance characteristics for parameter perturbations, external disturbances, and additive unmodeled dynamics.	Estimation is fundamentaly limited by the existence of external disturbances in the same frequency range as the reference input.
Design Complexity	The design is based upon well-known DAC principles which are straightforward state-space methods.	The controller design is based on one of two suggested algorithms. The resulting dynamical system is very complicated.	The compensation (controller) design and the estimator design are decoupled for the best results. Suitable recursive estimators are somewhat complicated.
Confidence of Operation (reliability)	The resulting controller is a linear, time-invariant (constant coefficient) system which can be analyzed using standard methods (i.e. Nyquist, Bode, Nichols, etc.).	The resulting controller is highly nonlinear and the only techniques available to gauge reliability are simulation and Liapanov functions.	The resulting closed-loop systems are nonlinear, and due to the decoupled nature of the estimator and control law development, are very difficult to analyze in general.

Figure 9.1. Relative Comparison of Adaptive Control Design Techniques

10.0 REFERENCES

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APPENDIX

Computer listings of controller evaluation simulations written in ACSL code.

A.1 Adaptive DAC (ADAC) Simulation

```
PROGRAM

INITIAL
      LOGICAL LDAC,LPULSE,LADAC,LTVWF
      CONSTANT A1IC=1,A2IC=2,AN1=1,AN2=2
      CONSTANT KIC=.5,BIC=1,KN=.5,BN=1
      CONSTANT MA1=Ø.,MA2=Ø.,MB=Ø.,MK=Ø.
      CONSTANT KO1=14.21, KO2=75.33
      CONSTANT KO3=351.5, KO4=-281.
      CONSTANT KO5=18., KO6=-782.
      CONSTANT X1IC=Ø., X2IC=Ø.
      CONSTANT XM1IC=Ø., XM2IC=Ø.
      CONSTANT CHIIC=Ø., E2HIC=Ø.
      CONSTANT ZA1IC=Ø., ZA2IC=Ø.
      CONSTANT Z1IC=Ø., Z2IC=Ø.
      CONSTANT PULSEP=2Ø.
      CONSTANT TSTOP =79.99,ISEMAX=1.,EMAX=.5
      CONSTANT WSINE=Ø.,WPULSE=Ø.,WNOISE=Ø.,WCONST=Ø.
      CONSTANT MWFREQ=Ø.1,WPULSP=4Ø.,WPULSW=2Ø.
      CONSTANT WMEAN=Ø.,WSTDEV=1.,NOISEW=3.
      CONSTANT LTVWF=.FALSE.
      CONSTANT LDAC=.TRUE.,LPULSE=.TRUE.,LADAC=.TRUE.
      PULSEW=PULSEP/2.
      U = Ø.
      CHI=Ø.
END $

DYNAMIC

DERIVATIVE
      CINTERVAL CINT = Ø.Ø5
      MAXTERVAL MAXT = Ø.Ø1
      NSTEPS = 1

"  INPUT  "

      DELTA = RSW(LPULSE,PULSE(Ø.,PULSEP,PULSEW),1.)

"  PARAMETER VARIATIONS  "

      A1  = MA1*T + A1IC
      A2  = MA2*T + A2IC
      B   = MB*T + BIC
      K   = MK*T + KIC

"  DISTURBANCE MODEL  "

      W = WSINE*SIN(RSW(LTVWF,2*3.1415*MWFREQ*T,2*3.1415*MWFREQ)*T) + ...
          WPULSE*PULSE(Ø.,WPULSP,WPULSW) + ...
          WNOISE*OU((1/NOISEW),WMEAN,WSTDEV) + ...
          WCONST

"  PLANT MODEL  "

      X1D = X2
      X1 = INTEG(X1D,X1IC)

      X2D = -A1*X1 - A2*X2 + U + W
      X2 = INTEG(X2D,X2IC)

      Y = K*((B/K)*X1 + X2)

"  OBSERVER  "
```

```
        E = DELTA - Y
        INOV = - Y - CHI

        CHID = E2H + KO1*INOV
        CHI = INTEG(CHID,CHIIC)

        E2HD = -AN1*CHI - AN2*E2H - AN1*DELTA + ZA1H - Z1H...
               -BN*(UA + UD) + KO2*INOV
        E2H = INTEG(E2HD,E2HIC)

        ZA1HD = ZA2H + KO3*INOV
        ZA1H = INTEG(ZA1HD,ZA1IC)

        ZA2HD = -AN1*ZA1H - AN2*ZA2H + KO4*INOV
        ZA2H = INTEG(ZA2HD,ZA2IC)

        Z1HD = Z2H + KO5*INOV
        Z1H = INTEG(Z1HD,Z1IC)

        Z2HD = KO6*INOV
        Z2H = INTEG(Z2HD,Z2IC)

        YH = -CHI

"   CONTROL     "

        UP = (AN1/BN)*DELTA
        UD = - RSW(LDAC,(1/BN)*Z1H,Ø.)
        UA =  RSW(LADAC,(1/BN)*ZA1H,Ø.)
        U = (UP + UA + UD)

" IDEAL MODEL  "

        XM1D = XM2
        XM1 = INTEG(XM1D,XM1IC)

        XM2D = -AN1*XM1 - AN2*XM2 + DELTA
        XM2 = INTEG(XM2D,XM2IC)

        YM = KN*((BN/KN)*XM1 + XM2)

" INTEGRAL SQUARED ERROR  "

        ERROR = YM - Y
        ISE = INTEG(ERROR**2,Ø.)

"  TERMINATION   "

        TERMT ((ISE .GE. ISEMAX) .OR. (ABS(EP      .GE. EMAX) ...
                  .OR. (T .GE. TSTOP))

END $"OF DERIVATIVE"

END $"OF DYNAMIC"

END
```

A.1 Polynomial DAC (ADAC) Simulation

```
 PROGRAM

INITIAL
        LOGICAL LDAC,LPULSE
        CONSTANT A1IC=1,A2IC=2,AN1=1,AN2=2
        CONSTANT KIC=.5,BIC=1,KN=.5,BN=1
        CONSTANT MA1=Ø.,MA2=Ø.,MB=Ø.,MK=Ø.
        CONSTANT KE1=1.846, KE2=28.73
        CONSTANT KE3=211.32, KE4=695.
        CONSTANT KE5=1154., KE6=781.
        CONSTANT X1IC=Ø., X2IC=Ø.
        CONSTANT XM1IC=Ø., XM2IC=Ø.
        CONSTANT X1HIC=Ø., X2HIC=Ø.
        CONSTANT Z1HIC=Ø., Z2HIC=Ø., Z3HIC=Ø., Z4HIC=Ø.
        CONSTANT PULSEP=2Ø.
        CONSTANT TSTOP =1ØØØØ.
        CONSTANT W=Ø.,LDAC=.TRUE.,LPULSE=.TRUE.
        PULSEW=PULSEP/2.
        U = Ø.
END $

DYNAMIC

DERIVATIVE
        CINTERVAL CINT = Ø.Ø5
        MAXTERVAL MAXT = Ø.Ø1
        NSTEPS = 1

"  INPUT  "

        DELTA = RSW(LPULSE,PULSE(Ø.,PULSEP,PULSEW),1.)

"  PARAMETER VARIATIONS  "

        A1  = MA1*T + A1IC
        A2  = MA2*T + A2IC
        B   = MB*T + BIC
        K   = MK*T + KIC

"  PLANT MODEL  "

        X1D = X2
        X1 = INTEG(X1D,X1IC)

        X2D = -A2*X2 - A1*X1 + U + W
        X2 = INTEG(X2D,X2IC)

        Y = B*X1 + K*X2

"   OBSERVER     "

        DUM = Y - BN*X1H - KN*X2H

        X1HD = X2H + KE1*DUM
        X1H = INTEG(X1HD,X1HIC)

        X2HD = -AN1*X1H - AN2*X2H + Z1H + U + KE2*DUM
        X2H = INTEG(X2HD,X2HIC)

        Z1HD = Z2H + KE3*DUM
        Z1H = INTEG(Z1HD,Z1HIC)

        Z2HD = Z3H + KE4*DUM
```

```
      Z2H = INTEG(Z2HD,Z2HIC)

      Z3HD = Z4H + KE5*DUM
      Z3H = INTEG(Z3HD,Z3HIC)

      Z4HD = KE6*DUM
      Z4H = INTEG(Z4HD,Z4HIC)

"   CONT  L     "

      U = RSW(LDAC,-Z1H+(1/BN)*DELTA,(1/BN)*DELTA)

" IDEAL MODEL  "

      XM1D = XM2
      XM1 = INTEG(XM1D,XM1IC)

      XM2D = -AN1*XM1 - AN2*XM2 + DELTA
      XM2 = INTEG(XM2D,XM2IC)

      YM = KN*((BN/KN)*XM1 + XM2)

" INTEGRAL SQUARED ERROR  "

      ERROR = YM - Y
      ISE = INTEG(ERROR**2,Ø.)

"  TERMINATION   "

      TERMT ((ISE .GE. ISEMAX) .OR. (ABS(ERROR) .GE. EMAX) ...
               .OR. (T .GE. TSTOP))

END $"OF DERIVATIVE"

END $"OF DYNAMIC"

END
```

A.3 MRAC Simulation

```
PROGRAM

INITIAL
        LOGICAL LADAPT,LPULSE,LERROR,LXMOD,LUM,LERRCH,LTVWF,LTVWNS
        CONSTANT A1IC=1,A2IC=2,AN1=1,AN2=2
        CONSTANT KIC=.5,BIC=1,KN=.5,BN=1
        CONSTANT MA1=Ø.,MA2=Ø.,MB=Ø.,MK=Ø.
        CONSTANT X1IC=Ø., X2IC=Ø.
        CONSTANT X1MIC=Ø., X2MIC=Ø.
        CONSTANT C1IC=Ø.,C2IC=Ø.,C3IC=Ø.,C4IC=Ø.
        CONSTANT TN=2Ø.,TNB=2Ø.
        CONSTANT PULSEP=2Ø.
        CONSTANT TSTOP =79.99, ISEMAX=1., EMAX=.1
        CONSTANT WSINE=Ø.,WPULSE=Ø.,WNOISE=Ø.,WCONST=Ø.
        CONSTANT MWFREQ=Ø.1,WPULSP=4Ø.,WPULSW=2Ø.
        CONSTANT WMEAN=Ø.,WSTDEV=1.,NOISEW=3.,MNOISE=.Ø5
        CONSTANT LTVWF=.FALSE.,LTVWNS=.FALSE.
        CONSTANT LADAPT=.TRUE.,LPULSE=.TRUE.,LERRCH=.TRUE.
        CONSTANT LERROR=.TRUE.,LXMOD=.TRUE.,LUM=.TRUE.
        PULSEW=PULSEP/2.
        U = Ø.
END $

DYNAMIC

DERIVATIVE
        CINTERVAL CINT = Ø.1
        MAXTERVAL MAXT = Ø.Ø1
        NSTEPS = 1

*   INPUT  *

        DELTA = RSW(LPULSE,PULSE(Ø.,PULSEP,PULSEW),1.)
        UM = DELTA

*   PARAMETER VARIATIONS  *

        A1 = MA1*T + A1IC
        A2 = MA2*T + A2IC
        B  = MB*T + BIC
        K  = MK*T + KIC

*   DISTURBANCE MODEL  *

        W = WSINE*SIN(RSW(LTVWF,MWFREQ*T,MWFREQ)*T) + ...
            WPULSE*PULSE(Ø.,WPULSP,WPULSW) + ...
            WNOISE*OU(RSW(LTVWNS,1/(MNOISE*T+.ØØØ1),1/NOISEW),...
                    WMEAN,WSTDEV) + ...
            WCONST

*   PLANT MODEL  *

        X1D = X2
        X1 = INTEG(X1D,X1IC)

        X2D = -A1*X1 - A2*X2 + U + W
        X2 = INTEG(X2D,X2IC)

        Y = B*X1 + K*X2

*   REFERENCE MODEL  *

        X1MD = X2M
```

```
      X1M = INTEG(X1MD,X1MIC)

      X2MD = -AN1*X1M - AN2*X2M + UM
      X2M = INTEG(X2MD,X2MIC)

      YM = BN*X1M + KN*X2M

*  ADAPTIVE CONTROLLER    *

      EY = YM - Y

      C1D = TN*RSW(LERROR,EY,1.)*EY
      C2D = TN*RSW(LERROR,EY,1.)*X1M
      C3D = TN*RSW(LERROR,EY,1.)*X2M
      C4D = TN*RSW(LERROR,EY,1.)*UM

      C1 = INTEG(C1D,C1IC)
      C2 = INTEG(C2D,C2IC)
      C3 = INTEG(C3D,C3IC)
      C4 = INTEG(C4D,C4IC)

      KE = TNB*RSW(LERROR,EY,1.)*EY + C1
      KC1 = TNB*RSW(LERROR,EY,1.)*X1M + C2
      KC2 = TNB*RSW(LERROR,EY,1.)*X2M + C3
      KU = TNB*RSW(LERROR,EY,1.)*UM + C4

      U = RSW(LADAPT,RSW(LERRCH,KE*EY,Ø.)...
              + RSW(LXMOD,KC1*X1M + KC2*X2M,Ø.)...
              + RSW(LUM,KU*UM,Ø.),UM)

*  TERMINATION   *

      ISE = INTEG(EY**2,Ø.)
      TERMT ((ISE .GE. ISEMAX) .OR. (ABS(EY) .GE. EMAX) ...
              .OR. (T .GE. TSTOP))

END $"OF DERIVATIVE"

END $"OF DYNAMIC"

END
```

A.4 PID Simulation

```
PROGRAM

INITIAL
        ARRAY QN(2),QD(2)
        LOGICAL LDAC,LPULSE,LADAC,LTVWF
        CONSTANT A1IC=1,A2IC=2,AN1=1,AN2=2
        CONSTANT KIC=.5,BIC=1,KN=.5,BN=1
        CONSTANT MA1=Ø.,MA2=Ø.,MB=Ø.,MK=Ø.
        CONSTANT KP=1., KI=1.1, KF=.55, KD=Ø.
        CONSTANT X1IC=Ø., X2IC=Ø.
        CONSTANT XM1IC=Ø., XM2IC=Ø.
        CONSTANT CHIIC=Ø., E2HIC=Ø.
        CONSTANT ZA1IC=Ø., ZA2IC=Ø.
        CONSTANT Z1IC=Ø., Z2IC=Ø.
        CONSTANT QN=1.,Ø.
        CONSTANT QD=Ø.1,1.
        CONSTANT PULSEP=2Ø.
        CONSTANT TSTOP =79.99,ISEMAX=1.3,EMAX=.1
        CONSTANT WSINE=Ø.,WPULSE=Ø.,WNOISE=Ø.,WCONST=Ø.
        CONSTANT MWFREQ=Ø.1,WPULSP=4Ø.,WPULSW=2Ø.
        CONSTANT WMEAN=Ø.,WSTDEV=1.,NOISEW=3.
        CONSTANT LDAC=.FALSE.,LPULSE=.FALSE.
        CONSTANT LADAC=.FALSE.,LTVWF=.FALSE.
        PULSEW=PULSEP/2.
        U = Ø.
        CHI=Ø.
END $

DYNAMIC

DERIVATIVE
        CINTERVAL CINT = Ø.Ø5
        MAXTERVAL MAXT = Ø.Ø1
        NSTEPS = 1

"  INPUT  "

        DELTA = RSW(LPULSE,PULSE(Ø.,PULSEP,PULSEW),1.)

"  PARAMETER VARIATIONS  "

        A1 = MA1*T + A1IC
        A2 = MA2*T + A2IC
        B  = MB*T + BIC
        K  = MK*T + KIC

"  DISTURBANCE MODEL  "

        W = WSINE*SIN(RSW(LTVWF,MWFREQ*T,MWFREQ)*T) + ...
            WPULSE*PULSE(Ø.,WPULSP,WPULSW) + ...
            WNOISE*OU((1/NOISEW),WMEAN,WSTDEV) + ...
            WCONST

"   CONTROL      "

        E = DELTA - Y
        U = KF*(KP*E + KI*INTEG(E,Ø.) + KD*TRAN(1,1,QN,QD,E))

"  PLANT MODEL  "

        X1D = X2
        X1 = INTEG(X1D,X1IC)
```

```
X2D = -A1*X1 - A2*X2 + U + W
X2 = INTEG(X2D,X2IC)

Y = K*((B/K)*X1 + X2)

' IDEAL MODEL '

XM1D = XM2
XM1 = INTEG(XM1D,XM1IC)

XM2D = -AN1*XM1 - AN2*XM2 + DELTA
XM2 = INTEG(XM2D,XM2IC)

YM = KN*((BN/KN)*XM1 + XM2)

' INTEGRAL SQUARED ERROR '

ERROR = YM - Y
ISE = INTEG(ERROR**2,Ø.)

'  TERMINATION  '

TERMT ((ISE .GE. ISEMAX) .OR. (ABS(ERROR) .GE. EMAX)...
       .OR. (T .GE. TSTOP))

END $'OF DERIVATIVE'

END $'OF DYNAMIC'

END
```

A.5 STR Simulation

```
PROGRAM

INITIAL
        ARRAY QNUY(4), QDUY(4), QNYREF(4), QDYREF(4)
        LOGICAL LPULSE,LSINE,LNOISE,LSTEP
        LOGICAL DMODEL,LCLOOP,TRUPAR,LGAIN
        CINTERVAL CINT = .5
        MAXTERVAL MAXT = .01
        NSTEPS NSTP = 1
        CONSTANT LAMBDA = 1.
        CONSTANT COVIC=100.
        CONSTANT TSTOP=24.99
        CONSTANT TSAMP=.01
        CONSTANT THT1IC=0., THT2IC=0.
        CONSTANT THT3IC=.5, THT4IC=1.
        CONSTANT PULSEP=20.,PULSEW=10.,FSINE=.25
        CONSTANT KPULSE=1.,KSINE=1.
        CONSTANT RMSN=.5
        CONSTANT A1IC=1,A2IC=2,AN1=1,AN2=2
        CONSTANT KIC=.5,BIC=1,KN=.5,BN=1
        CONSTANT MA1=0.,MA2=0.,MB=0.,MK=0.
        CONSTANT X1IC=0., X2IC=0.
        CONSTANT X1MIC=0., X2MIC=0.
        CONSTANT Y1IC=0., Y2IC=0.
        CONSTANT U1IC=0., U2IC=0.
        CONSTANT X1HIC=0., X2HIC=0.
        CONSTANT Z1HIC=0., Z2HIC=0., Z3HIC=0.
        CONSTANT W=0.
        CONSTANT LPULSE=.TRUE., LSINE=.FALSE.
        CONSTANT LNOISE=.FALSE., LSTEP=.FALSE.
        CONSTANT DMODEL=.FALSE., LCLOOP=.FALSE.,TRUPAR=.TRUE.
        CONSTANT LGAIN=.FALSE.
        PULSEW=PULSEP/2.
        YL1 = 0.
        YL2 = 0.
        UL1 = 0.
        UL2 = 0.
        UN = 0.
        UF = 0.
        UFL = 0.
        UY = 0.
        THT1HL = THT1IC
        THT2HL = THT2IC
        THT3HL = THT3IC
        THT4HL = THT4IC
        P11L = COVIC
        P22L = COVIC
        P33L = COVIC
        P44L = COVIC
        P12L = 0.
        P13L = 0.
        P14L = 0.
        P23L = 0.
        P24L = 0.
        P34L = 0.
        GSS=1.
        KH=KN
        BH=BN
        A1H=AN1
        A2H=AN2
END $

DYNAMIC
```

```
DERIVATIVE
"  PARAMETER VARIATIONS  "

        A1 = MA1*T + A1IC
        A2 = MA2*T + A2IC
        B  = MB*T + BIC
        K  = MK*T + KIC

" INPUT "

        YREF = RSW(LPULSE,KPULSE*(PULSE(Ø.,PULSEP,PULSEW)),1.) + ...
            RSW(LSINE,KSINE*SIN(2.*3.1415*FSINE*T),Ø.) + ...
            RSW(LSTEP,1.,Ø.)
PROCEDURAL
        QNYREF(1)=1.
        QNYREF(2)=A2H
        QNYREF(3)=A1H
        QDYREF(1)=1.
        QDYREF(2)=Ø.
        QDYREF(3)=Ø.
END
        U = TRAN(2,2,QNYREF,QDYREF,YREF) - RSW(LCLOOP,UY,Ø.)

"  PLANT MODEL  "

        X1D = X2
        X1 = INTEG(X1D,X1IC)

        X2D = -A1*X1 - A2*X2 + U + W
        X2 = INTEG(X2D,X2IC)

        Y = B*X1 + K*X2

"  FILTER Y FOR USE IN ESTIMATOR  "

        Y1D = Y2
        Y1 = INTEG(Y1D,Y1IC)

        Y2D = -AN2*Y1 - AN1*Y2 + Y
        Y2 = INTEG(Y2D,Y2IC)

        U1D = U2
        U1 = INTEG(U1D,U1IC)

        U2D = -AN2*U1 - AN1*U2 + U
        U2 = INTEG(U2D,U2IC)

"  PLANT MODEL  "

        X1MD = X2M
        X1M = INTEG(X1MD,X1MIC)

        X2MD = -AN1*X1M - AN2*X2M + YREF
        X2M = INTEG(X2MD,X2MIC)

        YM = BN*X1M + KN*X2M

" CONTROL LAW  "

PROCEDURAL
        QNUY(1)=2.
        QNUY(2)=1.+2*A2H
```

```
          QNUY(3)=A2+2*A1H
          QNUY(4)=A1H
          QDUY(1)=KH
          QDUY(2)=BH
          QDUY(3)=Ø.
          QDUY(4)=Ø.
END
          UY = TRAN(3,3,QNUY,QDUY,Y)

END $"OF DERIVATIVE"

DISCRETE
          INTERVAL TS=.Ø1

PROCEDURAL

" PARAMETER ESTIMATION USING RECURSIVE LEAST SQUARES ...
          WITH EXPONENTIAL FORGETTING "

          E = Y - (-THT1HL*Y2-THT2HL*Y1+THT3HL*U2+THT4HL*U1)
          W1 = -P11L*Y2-P12L*Y1+P13L*U2+P14L*U1
          W2 = -P12L*Y2-P22L*Y1+P23L*U2+P24L*U1
          W3 = -P13L*Y2-P23L*Y1+P33L*U2+P34L*U1
          W4 = -P14L*Y2-P24L*Y1+P34L*U2+P44L*U1
          DEN = -W1*Y2-W2*Y1+W3*U2+W4*U1+LAMBDA

          THT1HN = THT1HL + W1*E/DEN
          THT2HN = THT2HL + W2*E/DEN
          THT3HN = THT3HL + W3*E/DEN
          THT4HN = THT4HL + W4*E/DEN

          P11N = (P11L - (W1**2)/DEN)/LAMBDA
          P22N = (P22L - (W2**2)/DEN)/LAMBDA
          P33N = (P33L - (W3**2)/DEN)/LAMBDA
          P44N = (P44L - (W4**2)/DEN)/LAMBDA
          P12N = (P12L - (W1*W2)/DEN)/LAMBDA
          P13N = (P13L - (W1*W3)/DEN)/LAMBDA
          P14N = (P14L - (W1*W4)/DEN)/LAMBDA
          P23N = (P23L - (W3*W2)/DEN)/LAMBDA
          P24N = (P24L - (W4*W2)/DEN)/LAMBDA
          P34N = (P34L - (W3*W4)/DEN)/LAMBDA

          A2H = AN1 + THT1HL
          A1H = AN2 + THT2HL
          KH = THT3HL
          BH = THT4HL

          THT1HL = THT1HN
          THT2HL = THT2HN
          THT3HL = THT3HN
          THT4HL = THT4HN
          P11L = P11N
          P22L = P22N
          P33L = P33N
          P44L = P44N
          P12L = P12N
          P13L = P13N
          P14L = P14N
          P23L = P23N
          P24L = P24N
          P34L = P34N

 END $"OF PROCEDURAL"
```

```
END $"OF DISCRETE"
"  TERMINATION   "
        TERMT (T .GE. TSTOP)
END $"OF DYNAMIC"
END
```

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